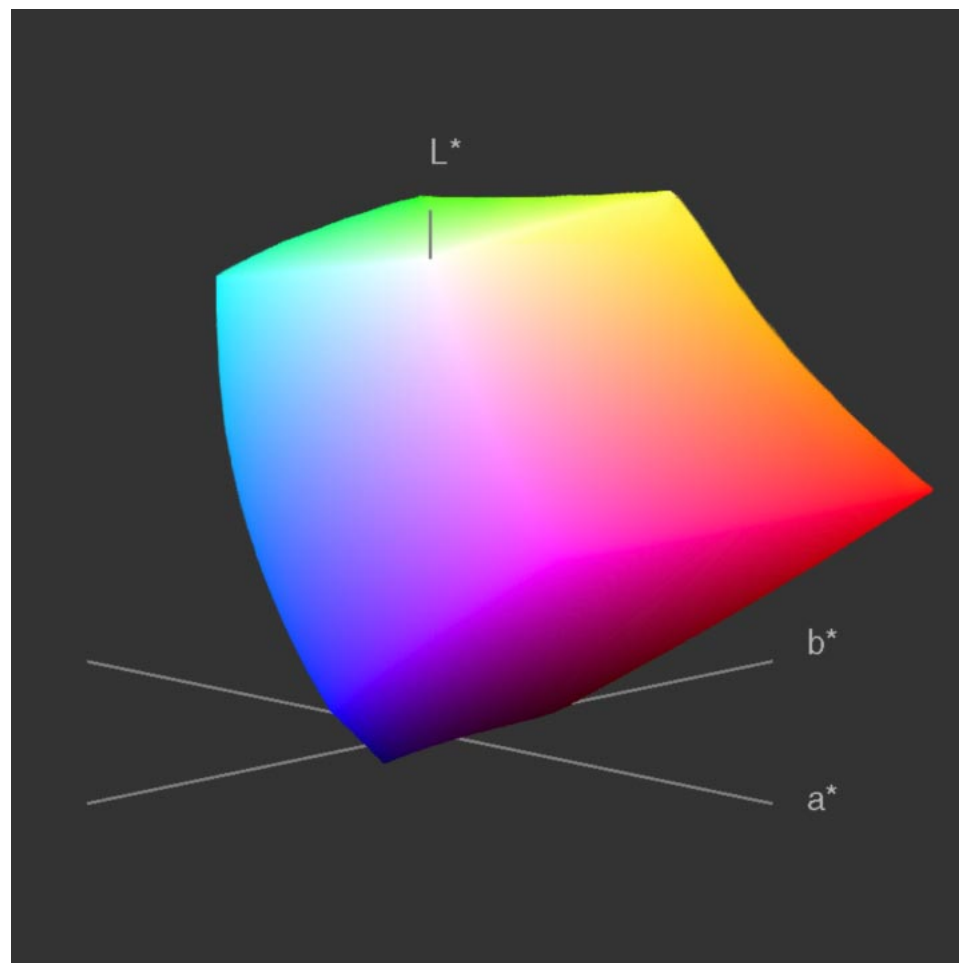


Gernot Hoffmann

CIELab Color Space



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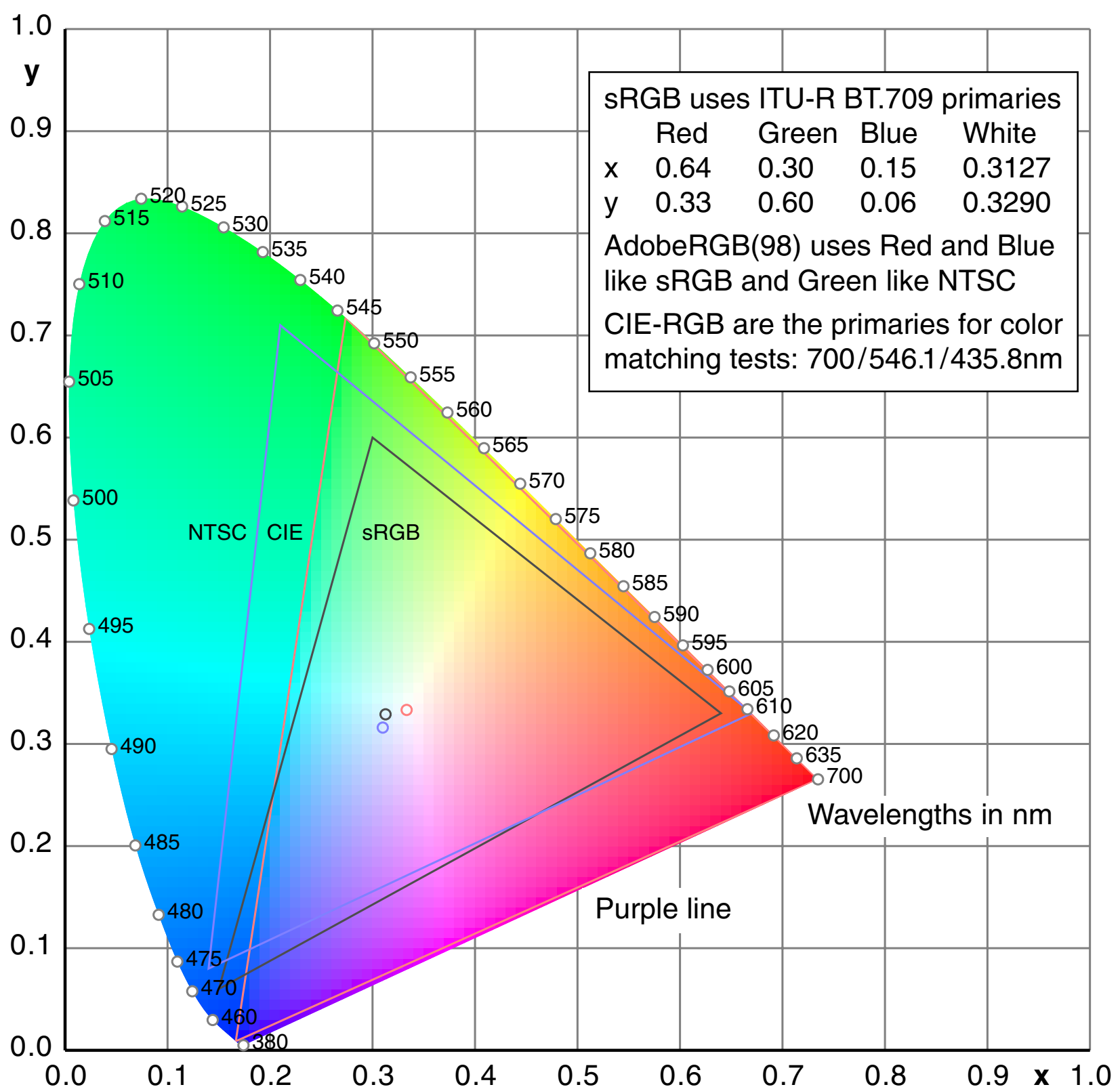
1.1 Introduction

The gamut for any RGB color order system is a triangle in the CIE xyY chromaticity diagram, here shown for the CIE primaries, the NTSC primaries and the Rec.709 primaries which are also valid for sRGB and therefore for many PC monitors. The white points are also different for all color spaces.

The CIE XYZ color space and its projection onto to the xy-plane, the CIE chromaticity diagram xyY, is often considered as perceptually not uniform. The colors in the diagram below are anyway only illustrations, but it's obvious that the green area looks exaggerated.

The CIELab color space was intended for equal perceptual differences for equal changes in the coordinates L^* , a^* and b^* . Color differences ΔE are defined as Euclidian distances in CIELab.

This document shows color charts for CIELab, referring to several RGB color spaces.



1.2 Introduction

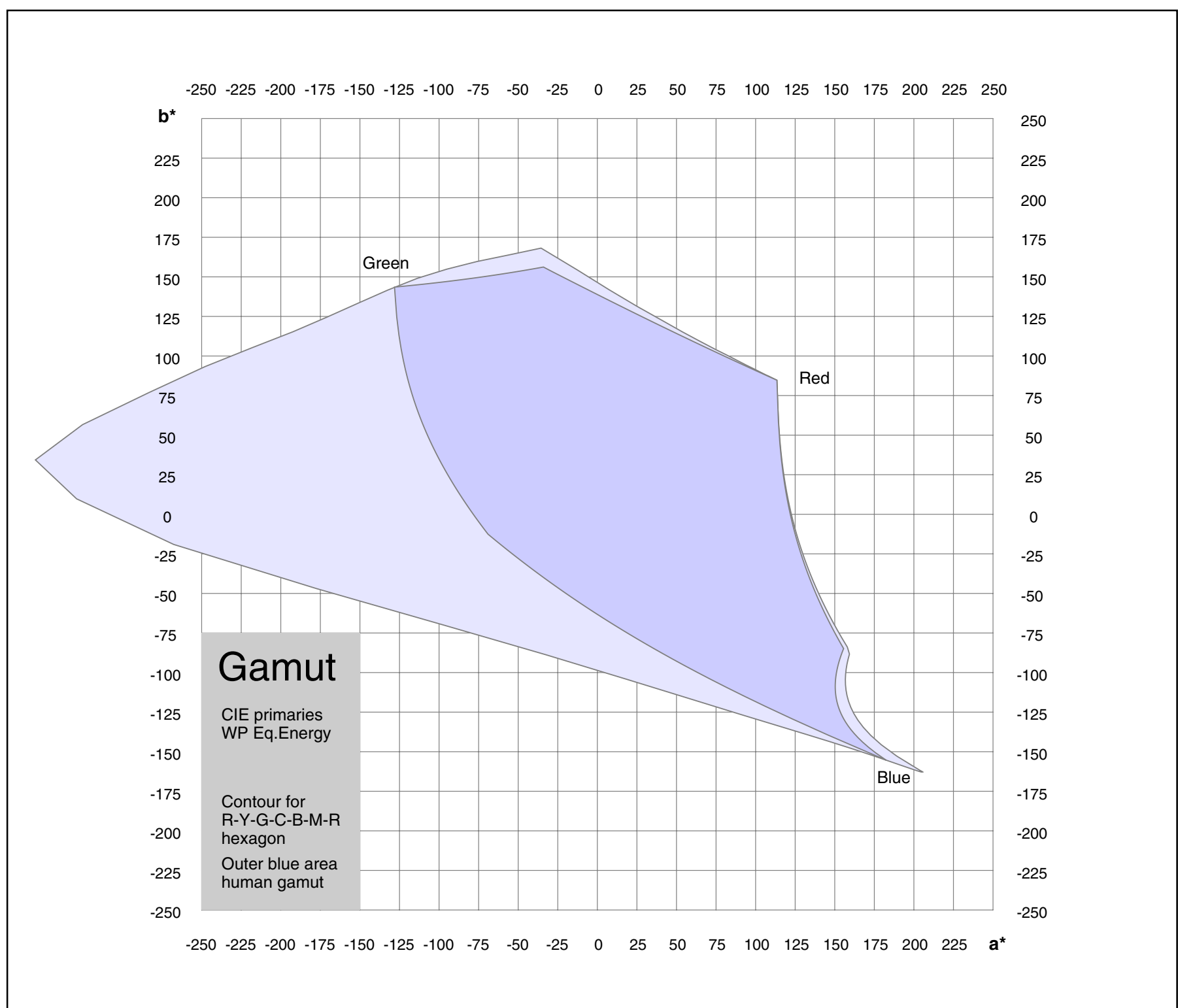
CIELab uses new coordinates L^* , a^* and b^* by functions as described in the next chapter. The true shape of the human gamut in CIELab is shown in chapter 12. This results in huge areas for higher luminance.

Just to show the orientation together with the gamut triangle for the primaries we proceed as follows:

Convert the contour points x, y for $Y=1$ into RGB. Divide R,G,B by the maximum of R,G,B. Convert R,G,B to L^*, a^*, b^* .

The gamut triangle in xyY has to be replaced by a representation of a color cube with corners R,G,B and Y(yellow),C,M. The gray axis is at $a^*=b^*=0$. Therefore the area is confined by a distorted hexagon. A similar diagram can be found in [1].

Later it will be shown that the actual gamut, which depends on the luminance, is a smaller area inside the hexagon.



2.1 Formulas

From R'G'B' to CIELab

R'G'B' are nonlinearly distorted values for each channel in physical tristimulus systems like CIE, NTSC or working spaces like sRGB or Adobe RGB(98).

RGB are undistorted values which are linearly related to CIE XYZ.

Each RGB system has a White Point(w). The transformation to CIELab requires a Reference White Point (n) which is either (w) or D50.

Issues of adaptation are taken into account by the Bradford correction.

For CIE and NTSC we use the same White Point in RGB and CIELab.

For sRGB we use D50 in CIELab and additionally the Bradford correction.

Generic gamma correction, $G=2.2$, $C=R,G,B$

$$C = C'^G$$

sRGB gamma correction, $C=R,G,B$

$$C = \begin{cases} C'/12.92 & \text{if } C' \leq 0.03928 \\ ((0.055 + C')/1.055)^{2.4} & \text{else} \end{cases}$$

RGB to XYZ (same white point D65)

$$\mathbf{X} = \mathbf{C}_{xr} \mathbf{R}$$

RGB to XYZ (new white point D50, Bradford correction)

$$\mathbf{X} = \mathbf{B} \mathbf{C}_{xr} \mathbf{R}$$

XYZ to $L^*a^*b^*$ (reference white \mathbf{X}_n)

$$X_1 = \frac{X}{X_n}$$

$$Y_1 = \frac{Y}{Y_n}$$

$$Z_1 = \frac{Z}{Z_n}$$

$$X_1 = \begin{cases} X_1^{1/3} & \text{if } X_1 > 0.008856 \\ 7.787 X_1 + 16/116 & \text{else} \end{cases}$$

$$Y_1 = \begin{cases} Y_1^{1/3} & \text{if } Y_1 > 0.008856 \\ 7.787 Y_1 + 16/116 & \text{else} \end{cases}$$

$$Z_1 = \begin{cases} Z_1^{1/3} & \text{if } Z_1 > 0.008856 \\ 7.787 Z_1 + 16/116 & \text{else} \end{cases}$$

$$L^* = 116 Y_1 - 16$$

$$a^* = 500 (X_1 - Y_1)$$

$$b^* = 200 (Y_1 - Z_1)$$

2.2 Formulas

From CIE Lab to R'G'B'

Please refer to explanations on the previous page.

A value RGB is out of gamut if any of the values is less than 0 or greater than 1 (normalized values).

L*a*b* to XYZ

$$Y_1 = (L^* + 16) / 116$$

$$X_1 = a^* / 500 + Y_1$$

$$Z_1 = -b^* / 200 + Y_1$$

$$X_1 = \begin{cases} X_1^3 & \text{if } X_1 > 0.206893 \\ (X_1 - 16/116) / 7.787 & \text{else} \end{cases}$$

$$Y_1 = \begin{cases} Y_1^3 & \text{if } Y_1 > 0.206893 \\ (Y_1 - 16/116) / 7.787 & \text{else} \end{cases}$$

$$Z_1 = \begin{cases} Z_1^3 & \text{if } Z_1 > 0.206893 \\ (Z_1 - 16/116) / 7.787 & \text{else} \end{cases}$$

$$X = X_n X_1$$

$$Y = Y_n Y_1$$

$$Z = Z_n Z_1$$

XYZ to RGB (same white point D50)

$$\mathbf{R} = \mathbf{C}_{rx} \mathbf{X} = \mathbf{C}_{xr}^{-1} \mathbf{X}$$

XYZ to RGB (new white point D65, Bradford correction)

$$\mathbf{R} = (\mathbf{B} \mathbf{C}_{xr})^{-1} \mathbf{X} = \mathbf{C}_{rx} \mathbf{B}^{-1} \mathbf{X}$$

Generic gamma correction, $G = 2.2$, $C = R, G, B$

$$C' = C^{1/G}$$

sRGB gamma correction, $C = R, G, B$

$$C' = \begin{cases} 12.92C & \text{if } C \leq 0.00304 \\ 1.055C^{1/2.4} - 0.055 & \text{else} \end{cases}$$

2.3 Formulas

From xyY to XYZ

The CIE chromaticity diagram is called xyY. The threedimensional space is called XYZ.

xyY to XYZ

$$\begin{aligned} z &= 1 - x - y \\ X &= Yx/y \\ Z &= Yz/y \end{aligned}$$

XYZ to xyY

$$\begin{aligned} D &= X + Y + Z \\ x &= X/D \\ y &= Y/D \\ z &= Z/D \end{aligned}$$

Some important numbers

$$\sqrt[3]{0.008856} = 0.2068930$$

$$16/116 = 0.137931$$

$$116 \cdot 0.008856^{1/3} - 16 = 903.3 \cdot 0.008856 = 8.0$$

2.4 Formulas

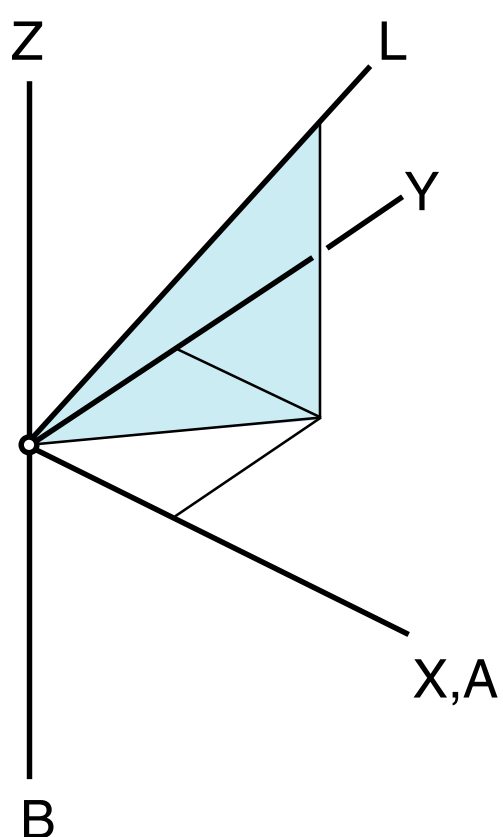
The conversion from XYZ to $L^*a^*b^*$ consists essentially of four steps:

1. White point correction
2. Nonlinear distortion of the variables X,Y,Z by a cubic root
3. Linear transformation into a new vector basis A,B,L
4. Scaling

With some obvious simplifications the linear part can be written by matrices:

$$\begin{bmatrix} A \\ B \\ L \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} A \\ B \\ L \end{bmatrix}$$



A is in X-direction, B in negative Z-direction and L along the diagonal in XYZ. A, B and L are normalized substitutes for a^* , b^* and L^* .

2.5 Formulas

Matrices for Primaries and White Point [3]

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \mathbf{C}_{xr} \begin{bmatrix} R \\ G \\ B \end{bmatrix}$$

$$\begin{bmatrix} R \\ G \\ B \end{bmatrix} = \mathbf{C}_{rx} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

$$D = (x_r - x_b)(y_g - y_b) - (y_r - y_b)(x_g - x_b)$$

$$U = (x_w - x_b)(y_g - y_b) - (y_w - y_b)(x_g - x_b)$$

$$V = (x_r - x_b)(y_w - y_b) - (y_r - y_b)(x_w - x_b)$$

$$u = U/D$$

$$v = V/D$$

$$w = 1 - u - v$$

$$\mathbf{C}_{xr} = \begin{bmatrix} u x_r/y_w & v x_g/y_w & w x_b/y_w \\ u y_r/y_w & v y_g/y_w & w y_b/y_w \\ u z_r/y_w & v z_g/y_w & w z_b/y_w \end{bmatrix}$$

$$\mathbf{C}_{rx} = \mathbf{C}_{xr}^{-1}$$

In the next formulas use c_{xrik} for c_{ik}

$$D = c_{11}(c_{22}c_{33} - c_{23}c_{32}) - c_{12}(c_{21}c_{33} - c_{23}c_{31}) + c_{13}(c_{21}c_{32} - c_{22}c_{31})$$

$$c_{rx11} = (c_{22}c_{33} - c_{23}c_{32})/D$$

$$c_{rx12} = (c_{12}c_{33} - c_{13}c_{32})/D$$

$$c_{rx13} = (c_{12}c_{23} - c_{13}c_{22})/D$$

$$c_{rx21} = (c_{21}c_{33} - c_{23}c_{31})/D$$

$$c_{rx22} = (c_{11}c_{33} - c_{13}c_{31})/D$$

$$c_{rx23} = (c_{11}c_{23} - c_{13}c_{21})/D$$

$$c_{rx31} = (c_{21}c_{32} - c_{22}c_{31})/D$$

$$c_{rx32} = (c_{11}c_{32} - c_{12}c_{31})/D$$

$$c_{rx33} = (c_{11}c_{22} - c_{12}c_{21})/D$$

3.1 Primaries and Matrices

CIE Primaries and Equi-Energy Whitepoint					
x_r	y_r	x_g	y_g	x_b	y_b
0.73467	0.26533	0.27376	0.71741	0.16658	0.00886
x_w	y_w	x_w	y_w	z_w	
0.33333	0.33333	1	1	1	
Matrix $\mathbf{X} = \mathbf{C}_{xr} \mathbf{R}$			Matrix $\mathbf{R} = \mathbf{C}_{rx} \mathbf{X}$		
0.4900	0.3100	0.2000	+2.3647	−0.8966	−0.4681
0.1770	0.8124	0.0106	−0.5152	+1.4264	+0.0887
0.0000	0.0100	0.9900	+0.0052	−0.0144	+1.0092

NTSC Primaries and NTSC WP (Illuminant C, 6774K)					
x_r	y_r	x_g	y_g	x_b	y_b
0.6700	0.3300	0.2100	0.7100	0.1400	0.0800
x_w	y_w	x_w	y_w	z_w	
0.3100	0.3160	0.9810	1	1.1835	
Matrix $\mathbf{X} = \mathbf{C}_{xr} \mathbf{R}$			Matrix $\mathbf{R} = \mathbf{C}_{rx} \mathbf{X}$		
0.6070	0.1734	0.2006	+1.9097	−0.5324	−0.2882
0.2990	0.5864	0.1146	−0.9850	+1.9998	+0.0283
0.0000	0.0661	1.1175	+0.0582	−0.1182	+0.8966

sRGB / Rec.709 Primaries and D65					
x_r	y_r	x_g	y_g	x_b	y_b
0.6400	0.3300	0.3000	0.6000	0.1500	0.0600
x_w	y_w	x_w	y_w	z_w	
0.3127	0.3290	0.9505	1	1.0891	
Matrix $\mathbf{X} = \mathbf{C}_{xr} \mathbf{R}$			Matrix $\mathbf{R} = \mathbf{C}_{rx} \mathbf{X}$		
0.4124	0.3576	0.1805	+3.2410	−1.5374	−0.4986
0.2126	0.7152	0.0722	−0.9692	+1.8760	+0.0416
0.0193	0.1192	0.9505	+0.0556	−0.2040	+1.0570

AdobeRGB(98) and D65					
x_r	y_r	x_g	y_g	x_b	y_b
0.6400	0.3300	0.2100	0.7100	0.1500	0.0600
x_w	y_w	x_w	y_w	z_w	
0.3127	0.3290	0.9505	1	1.0891	
Matrix $\mathbf{X} = \mathbf{C}_{xr} \mathbf{R}$			Matrix $\mathbf{R} = \mathbf{C}_{rx} \mathbf{X}$		
0.5767	0.1856	0.1882	+2.0416	−0.5650	−0.3447
0.2973	0.6274	0.0753	−0.9692	+1.8670	+0.0416
0.0270	0.0707	0.9913	+0.0134	−0.1184	+1.0152

Bradford Matrix for D65/D50					
Matrix \mathbf{B}			Matrix \mathbf{B}^{-1}		
+1.0479	+0.0229	−0.0502	+0.9555	−0.0231	+0.0633
+0.0296	+0.9904	−0.0171	−0.0283	+1.0100	+0.0211
−0.0092	+0.0151	+0.7519	+0.0123	−0.0206	+1.3303

3.2 Primaries and Matrices

This are the coordinates of the Primaries and the White Point in CIE XYZ for several cases (minor deviations to [1],[2] because of PostScript matrix inversion).

Prims	White	Y	Xw	Zw	Xr	Zr	Xg	Zg	Xb	Zb
CIE	EquiE	1.0000	1.0000	1.0003	2.7688	0.0000	0.3815	0.0123	18.8013	93.0654
CIE	NTSC	1.0000	0.9810	1.1835	2.7688	0.0000	0.3815	0.0123	18.8013	93.0654
CIE	D50	1.0000	0.9642	0.8251	2.7688	0.0000	0.3815	0.0123	18.8013	93.0654
CIE	D65	1.0000	0.9504	1.0890	2.7688	0.0000	0.3815	0.0123	18.8013	93.0654
CIE	9300K	1.0000	0.9714	1.4287	2.7688	0.0000	0.3815	0.0123	18.8013	93.0654
NTSC	NTSC	1.0000	0.9810	1.1835	2.0303	0.0000	0.2957	0.1126	1.7500	9.7500
709	D65	1.0000	0.9504	1.0890	1.9393	0.0909	0.5000	0.1666	2.5000	13.1666
WideG	D50	1.0000	0.9642	0.8251	2.7693	0.0000	0.1393	0.0706	8.8689	46.7625

4. Gamut Restrictions and Tests

Mostly it's assumed that all colors inside the gamut triangle can be shown by the respective device. Actually the gamut in CIELab is not the triangle R-G-B but the hexagon R-Y-G-C-B-M-R. We have to choose an interpolation path which shows also fully saturated yellow, cyan and magenta.

In the next diagrams we can see color patches which are out of gamut, though they are inside the hexagon. The gamut in the chromaticity diagram is the projection of all available colors in XYZ onto xyY, ignoring the spacial distribution in XYZ.

The available gamut depends strongly on the luminance Y.

A color is considered as out of gamut if at least one value of R,G,B is larger than one or smaller than zero. The values are clipped for these limits then.

Such a color is usually shown by the device reasonably, though slightly wrong.

The sRGB color charts were tested by Photoshop 7.

E.g. the chart sRGB-050 for L*=50. Select sRGB as working space and Rendering Intent Relative Colorimetric. Open document in RGB mode. Place page of PDF. Indicate CIELab values by info palette.

The nomenclature according to [8] is confusing:

Media-Relative = CIE Absolute

ICC-Absolute = CIE Relative

Photoshop Relative Colorimetric seems to mean ICC Media-Relative Colorimetric.

New tables Wide Gamut (October 17, 2003):
For Wide Gamut use this working space in Photoshop.

5. Inverse Gamma Correction

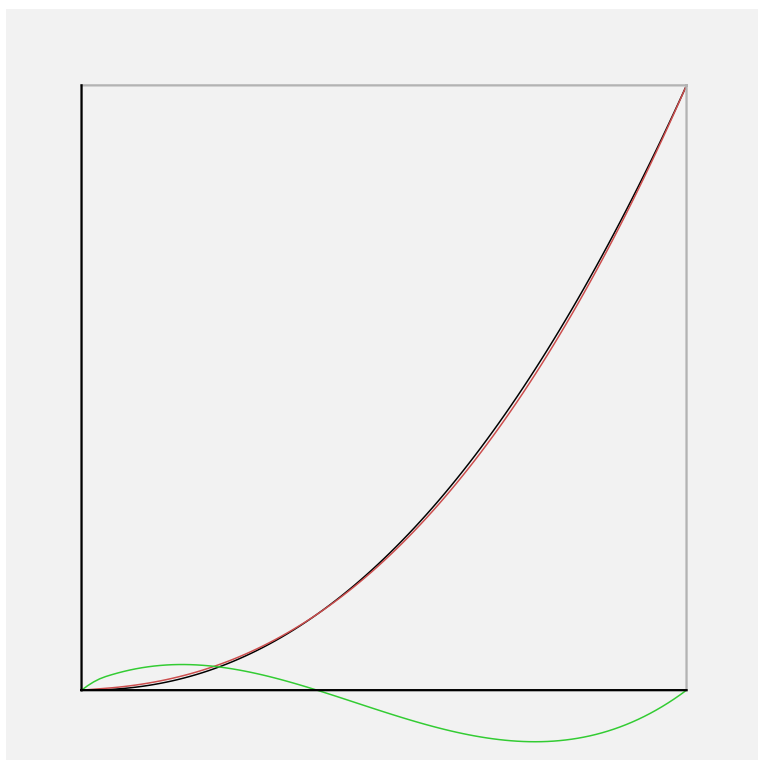
RGB values are transformed into nonlinear values $R'G'B'$ by $C' = C^{1/2.2}$ for CIE and NTSC primaries. This is a compensation for calibrated monitors.

sRGB differs slightly from Gamma=2.2 because the transfer function is composed of a linear part and a Gamma=2.4 part.

$$\begin{aligned} C' &= 1.055 C^{1/2.4} - 0.055 & \text{if } C > 0.00304 \\ C' &= 12.92 C & \text{else} \end{aligned}$$

The maximal difference is less than 1% full scale.

The diagram shows the Gamma corrections instead of the inverse corrections.



Black $C = C'^{2.2}$

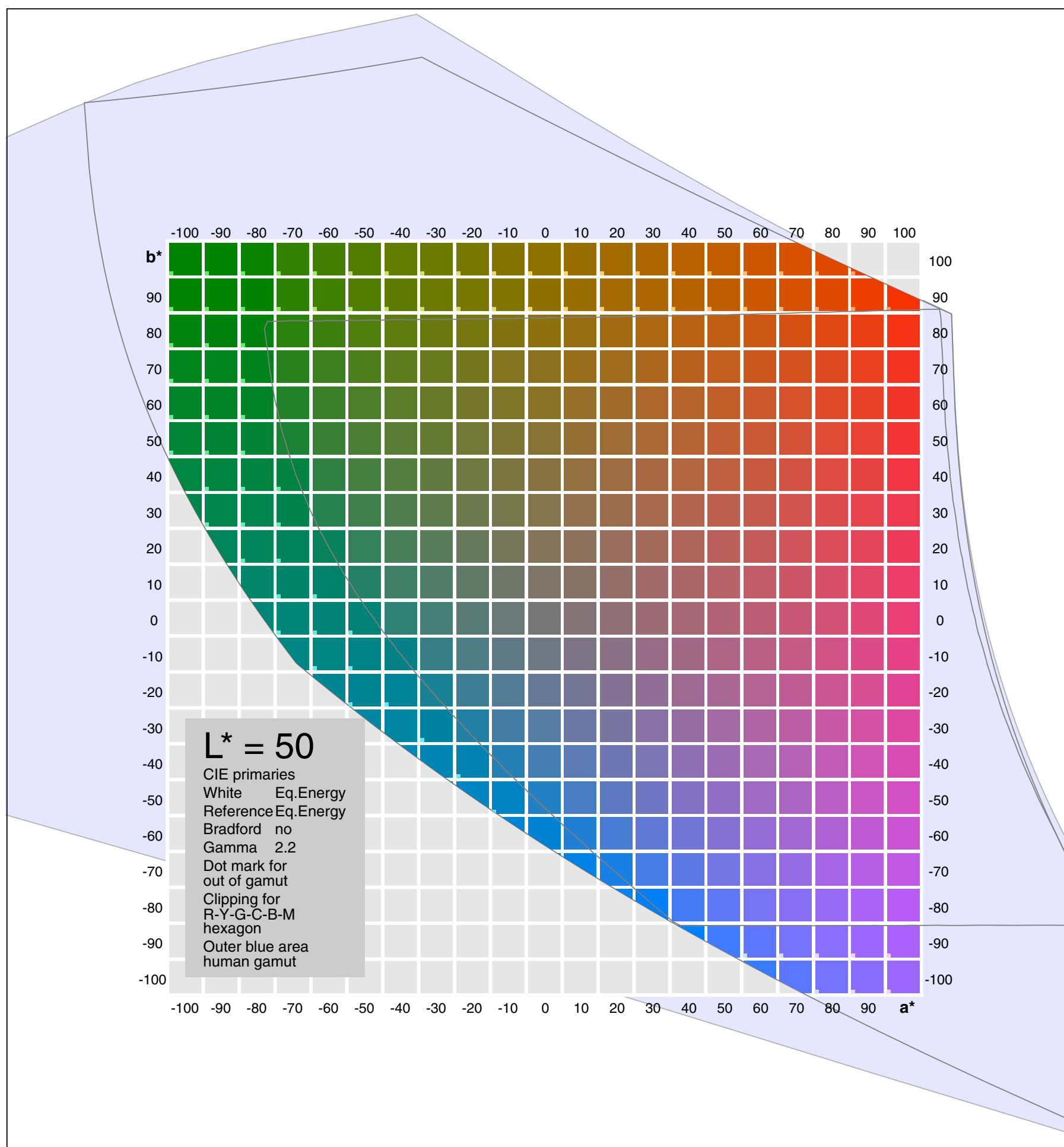
Red sRGB

Green Ten times the difference

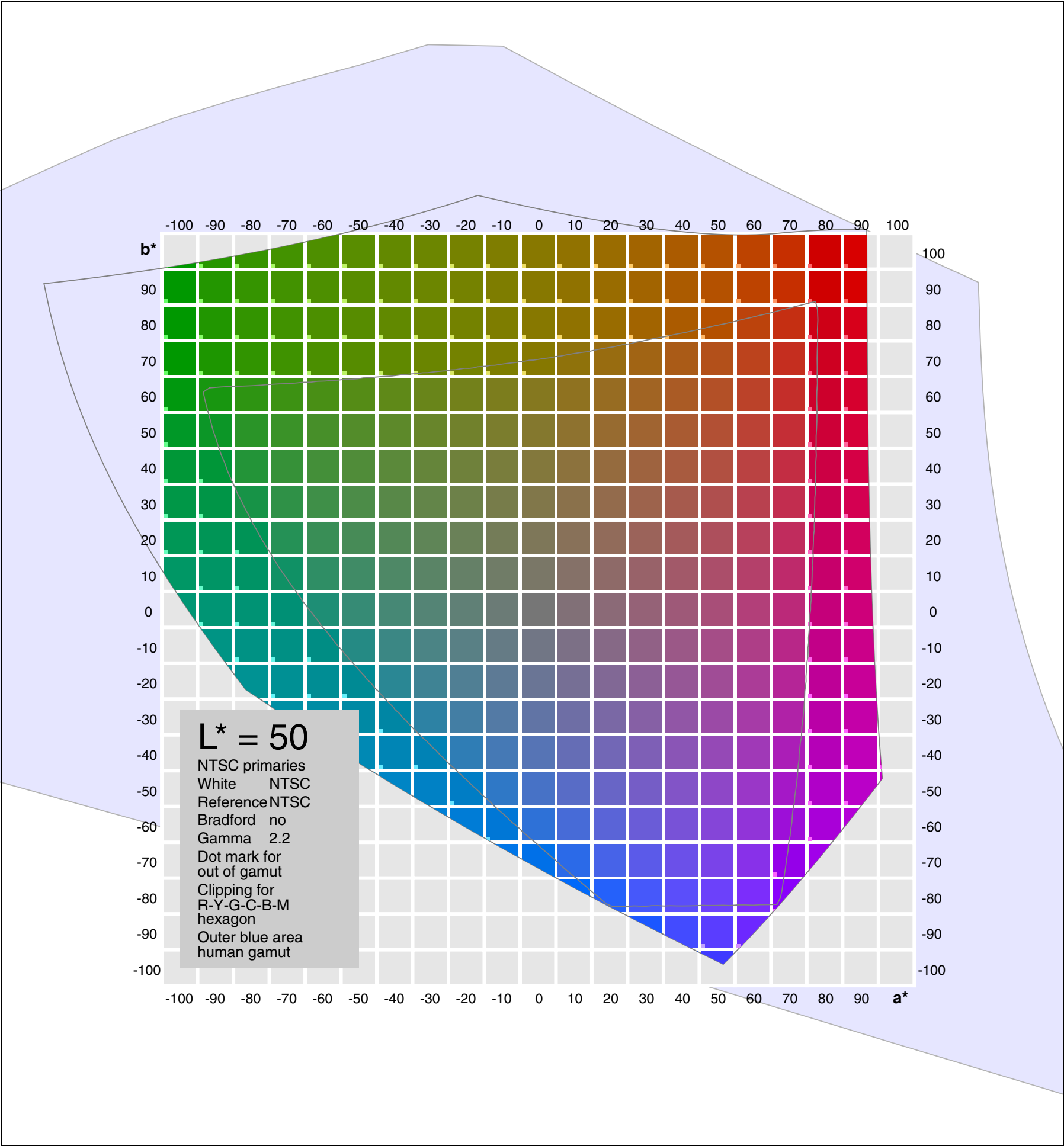
According to profile informations, NTSC and CIE-RGB seems to use the simple Gamma correction without linear part. Consequently we don't apply the sRGB formula here.

6. CIE $L^*=50$

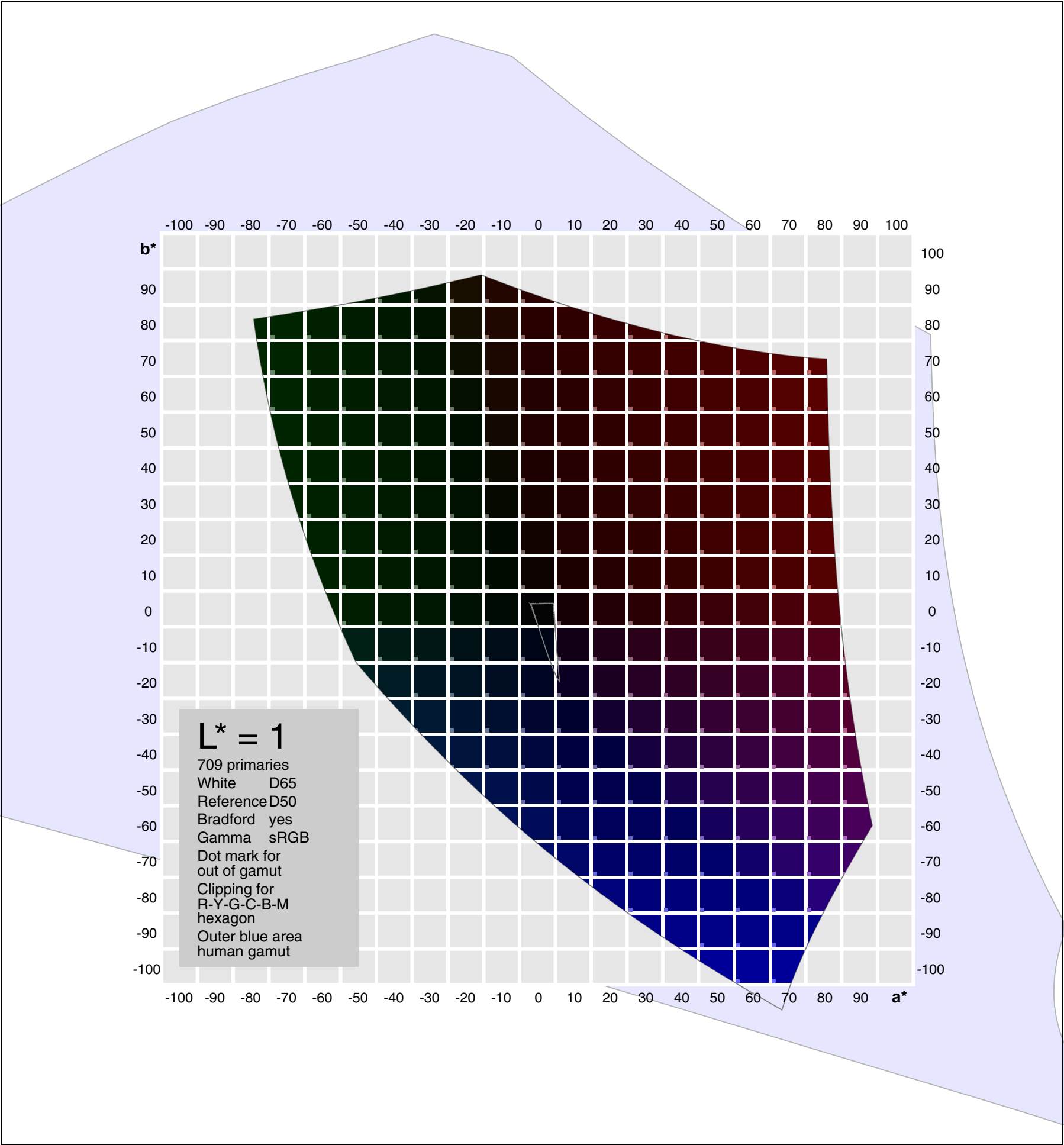
The illustrations are made by PostScript EPS programs as vector graphics.
Converted to PDF by Acrobat Distiller they are still vector graphics
Eventually the appearance can be improved by Smooth Line Art = Off



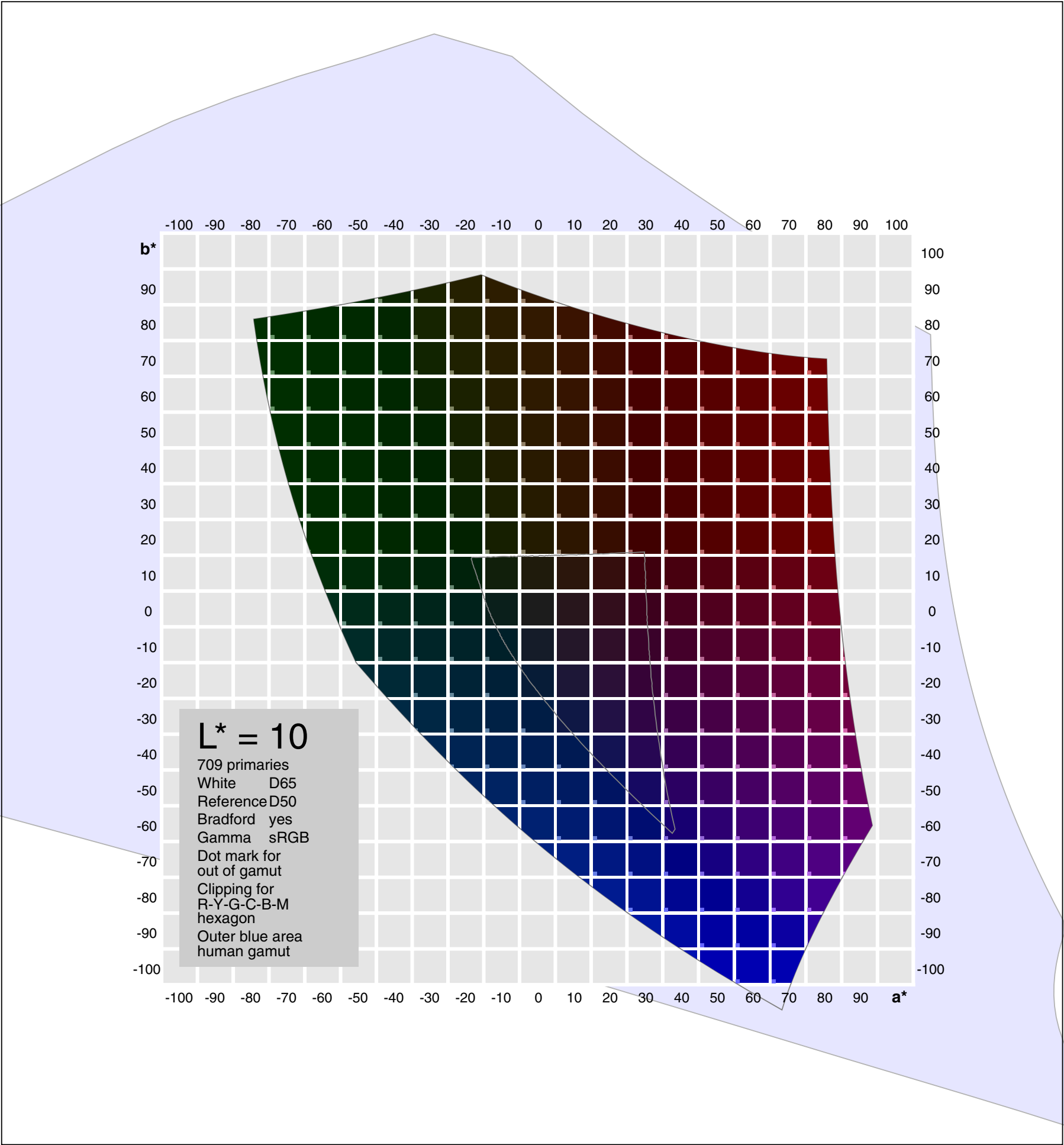
7. NTSC $L^*=50$



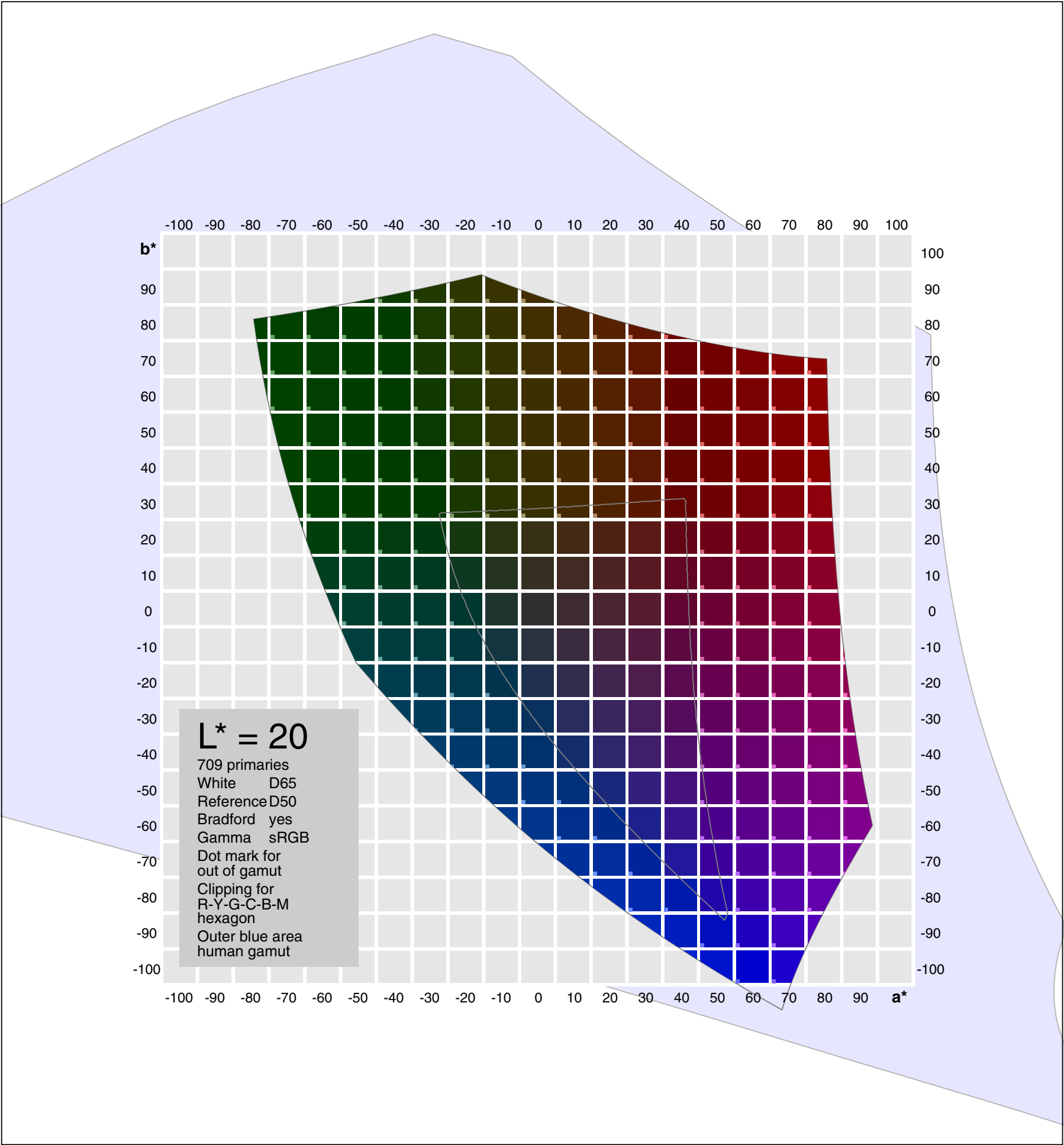
8.1 sRGB L*=1



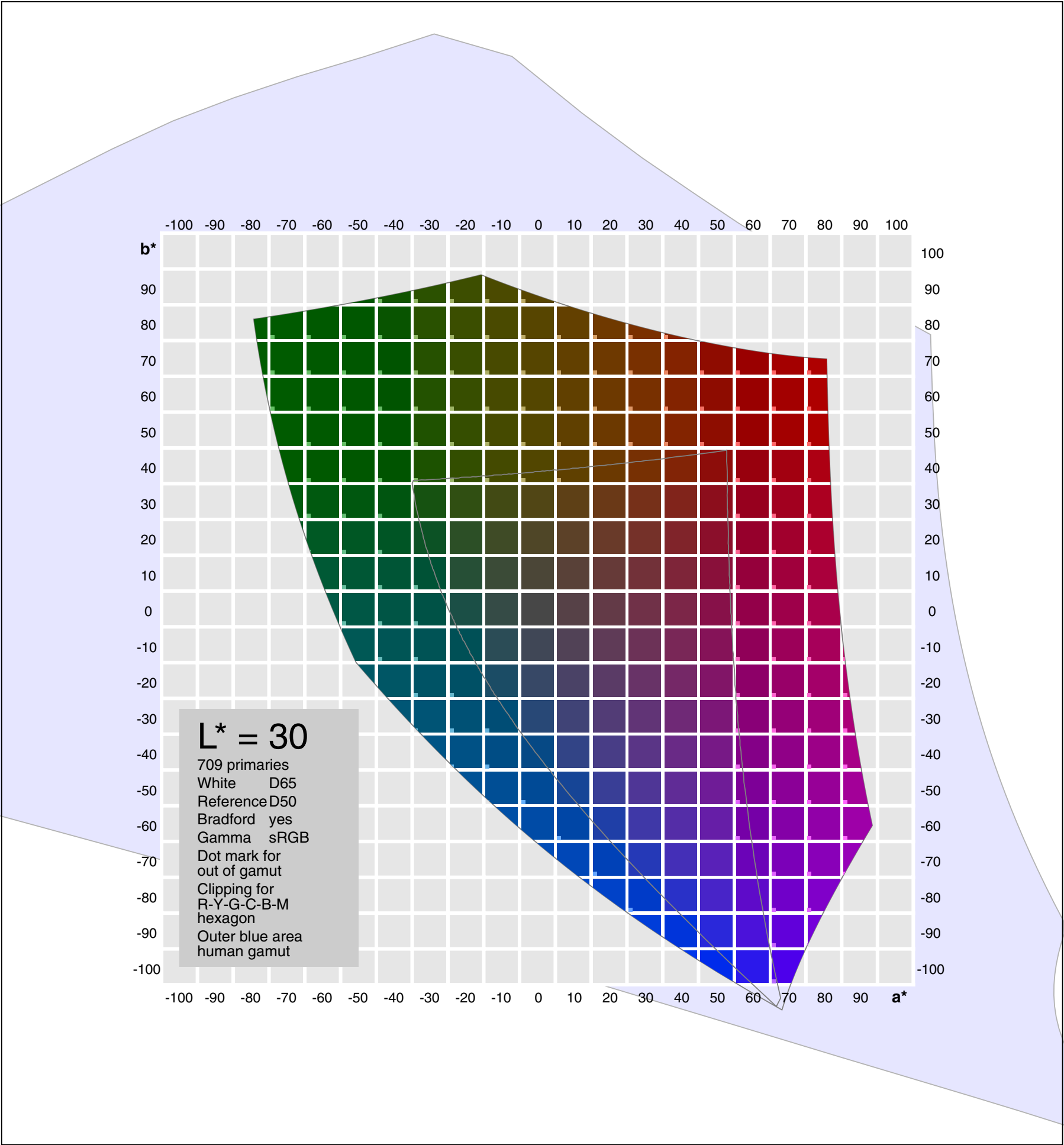
8.2 sRGB L*=10



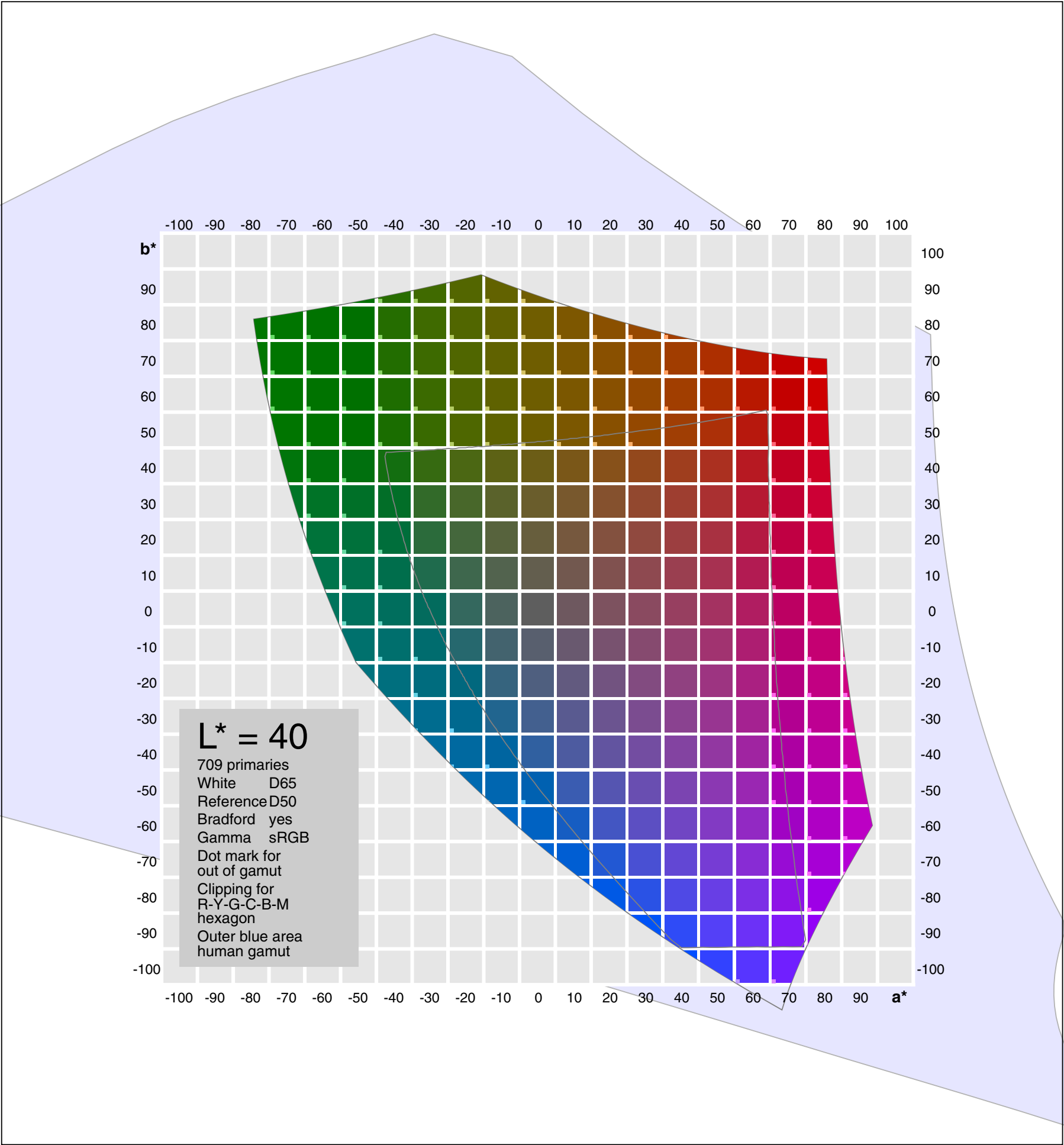
8.3 sRGB $L^*=20$



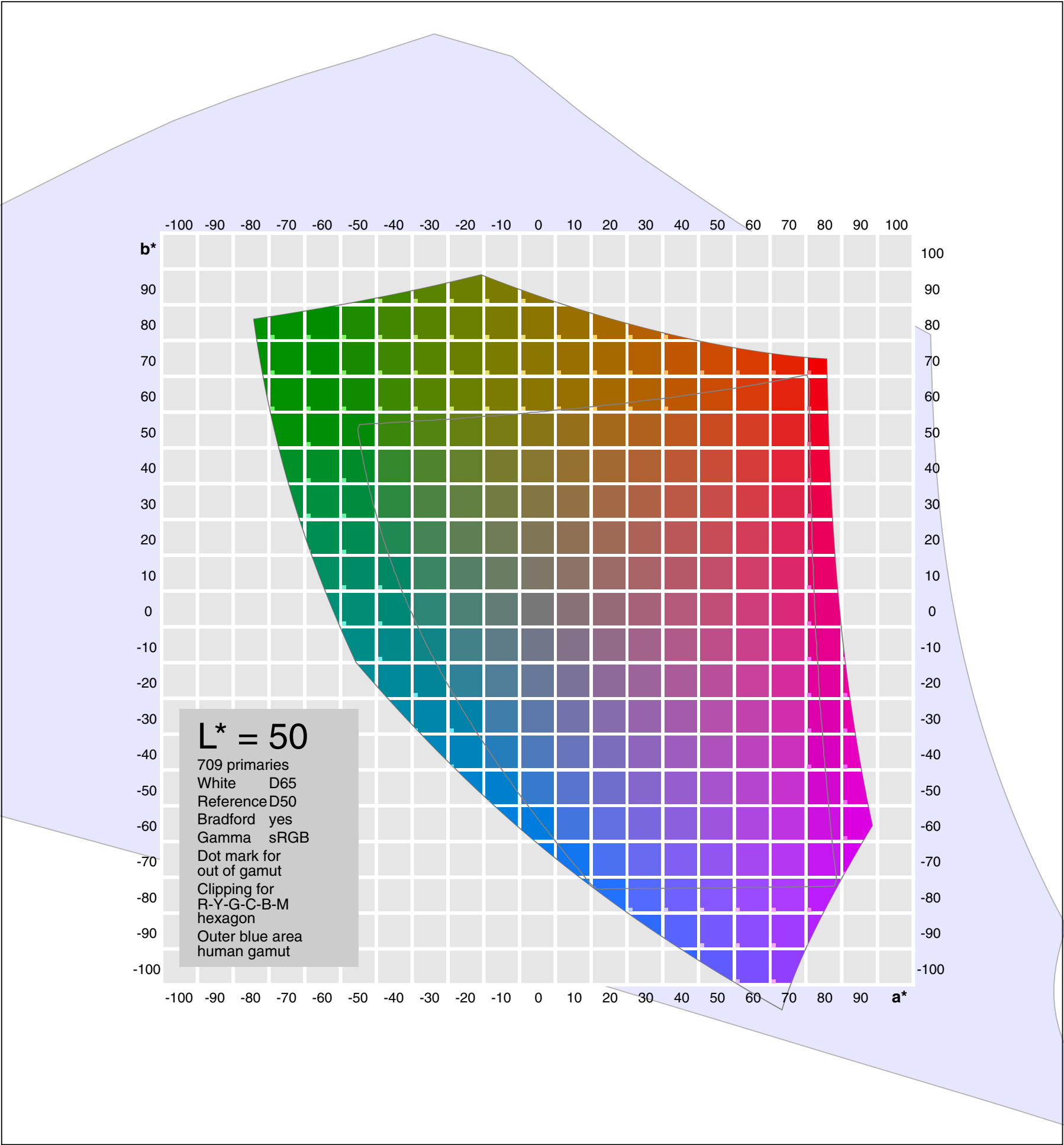
8.4 sRGB L*=30



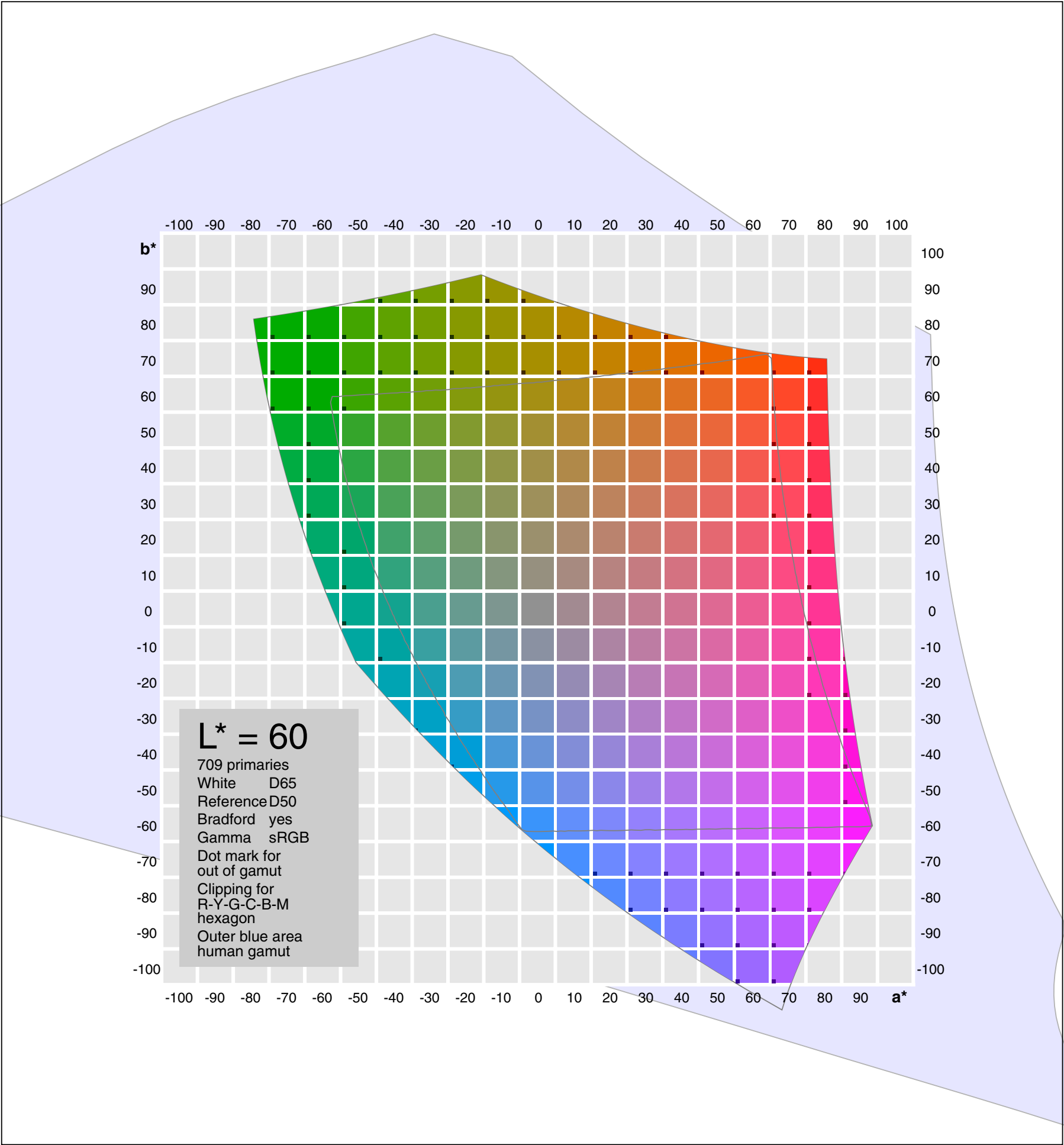
8.5 sRGB L*=40



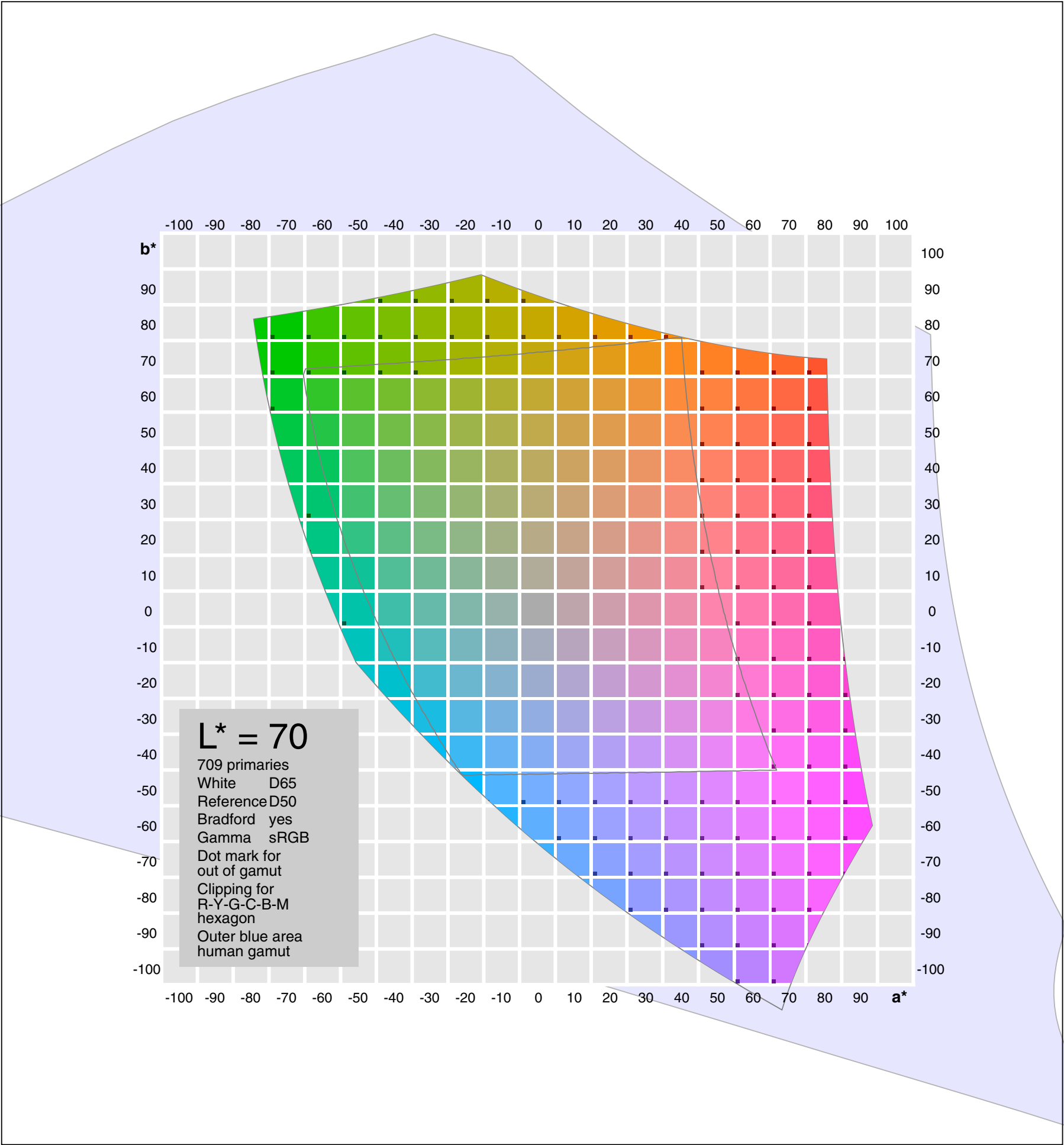
8.6 sRGB L*=50



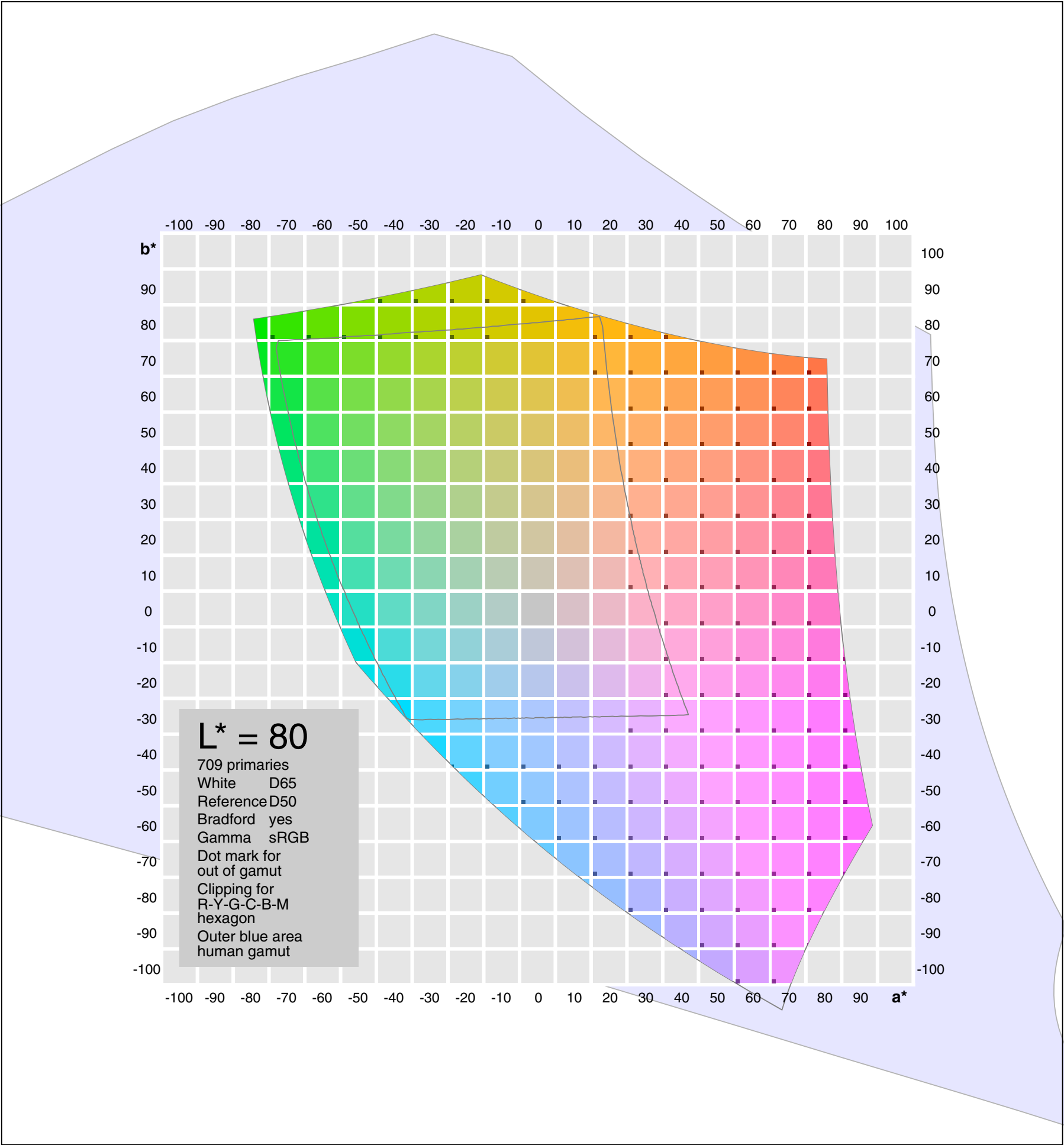
8.7 sRGB L*=60



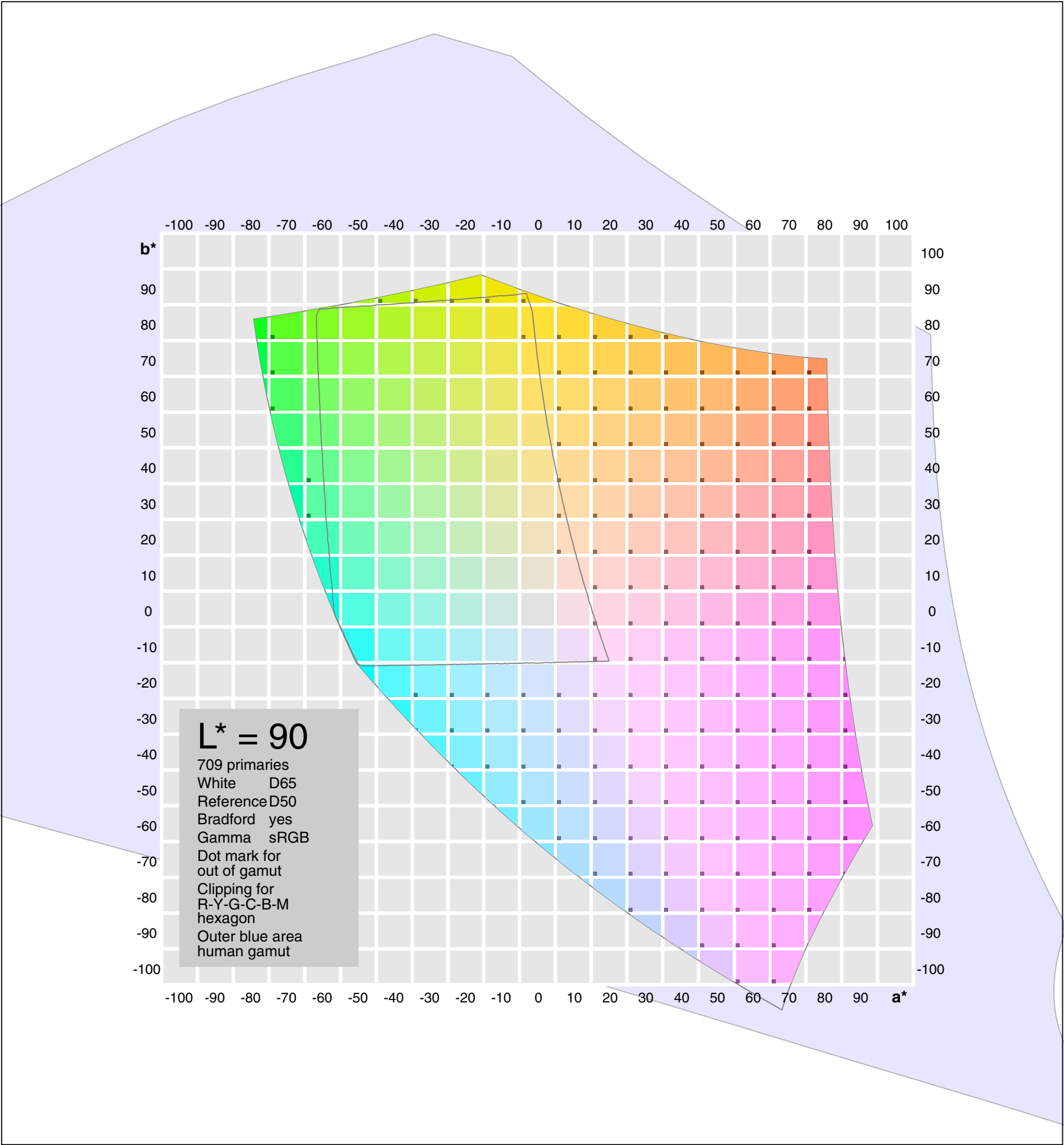
8.8 sRGB L*=70



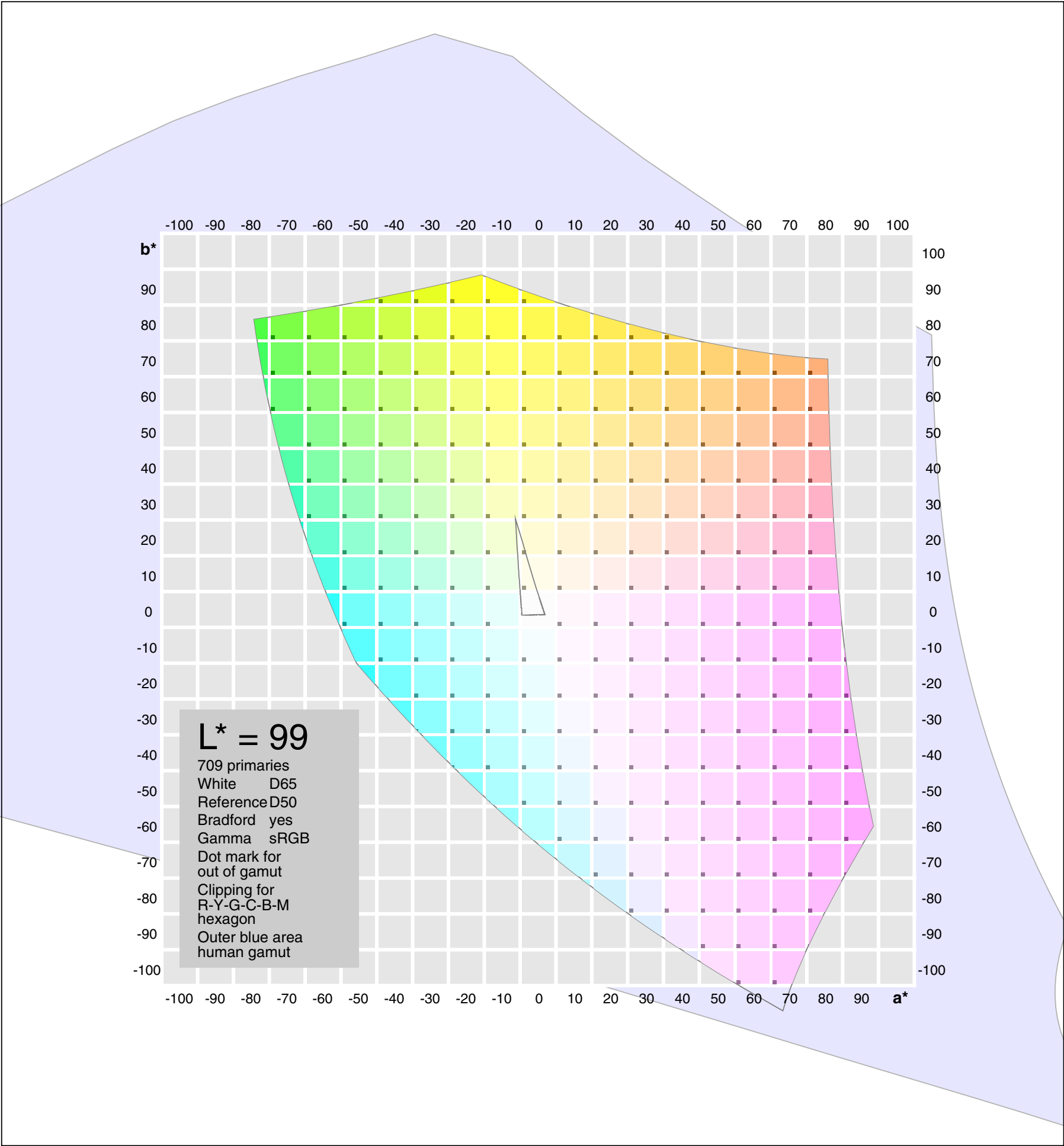
8.9 sRGB $L^*=80$



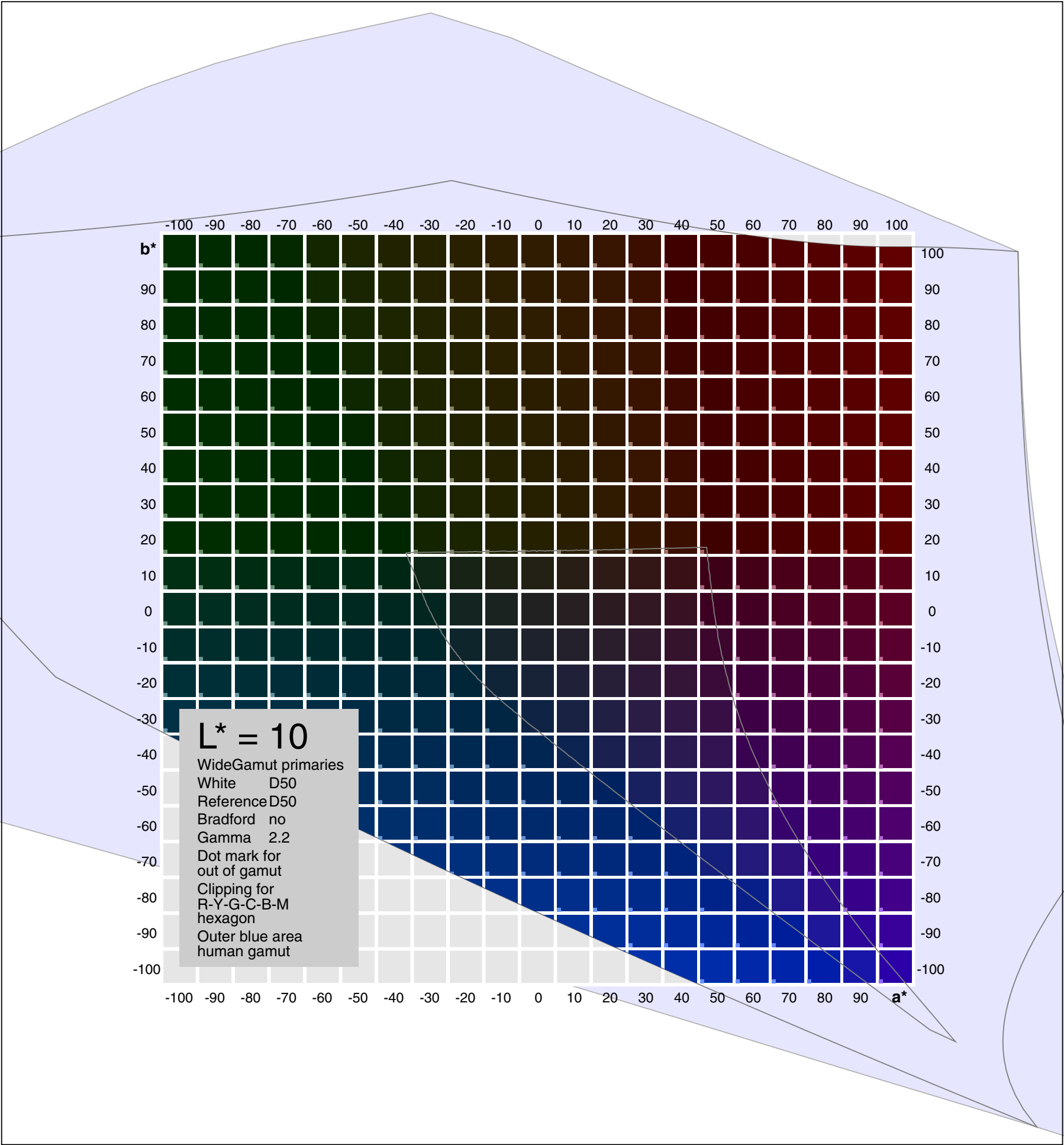
8.10 sRGB L*=90



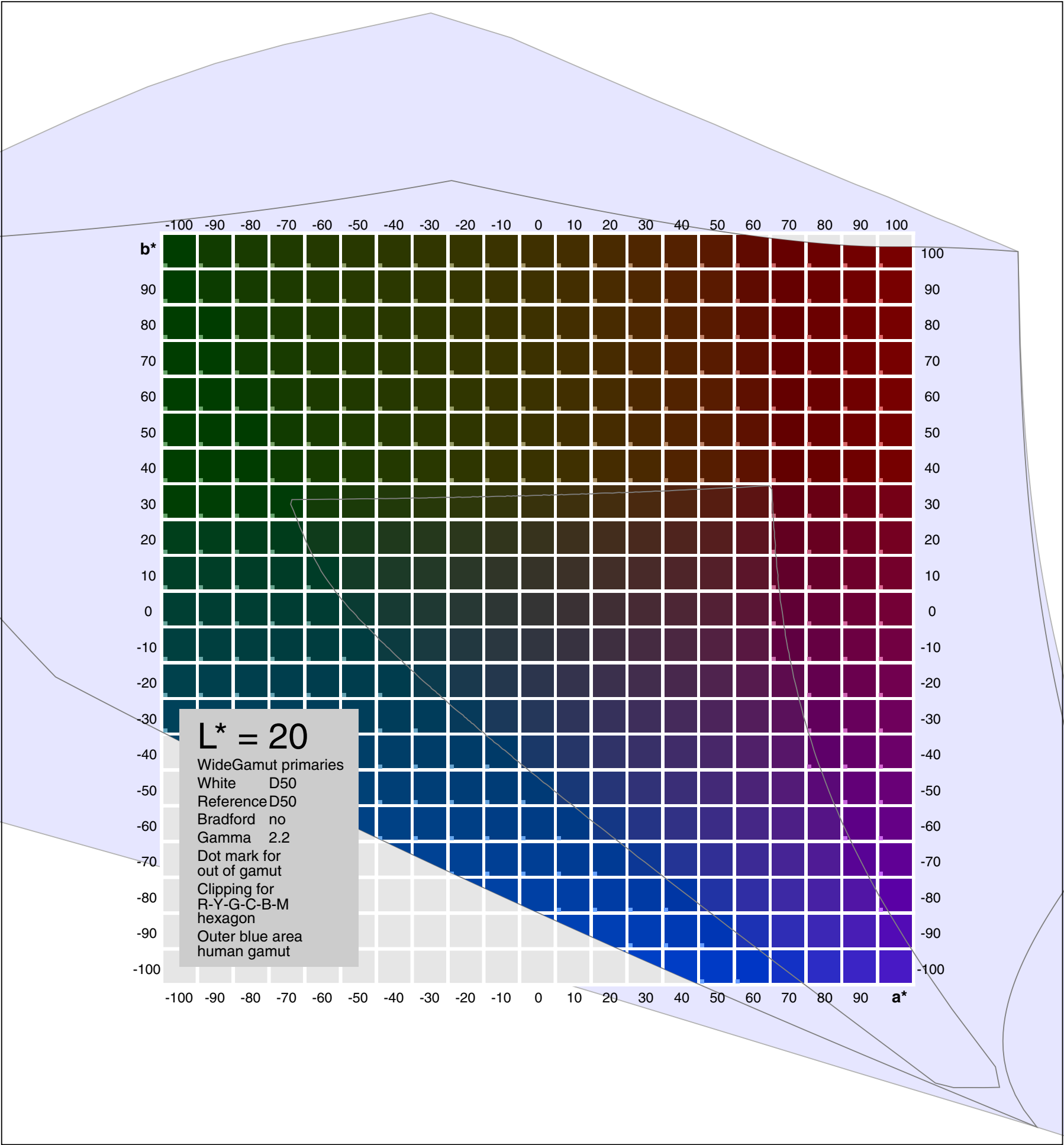
8.11 sRGB L*=99



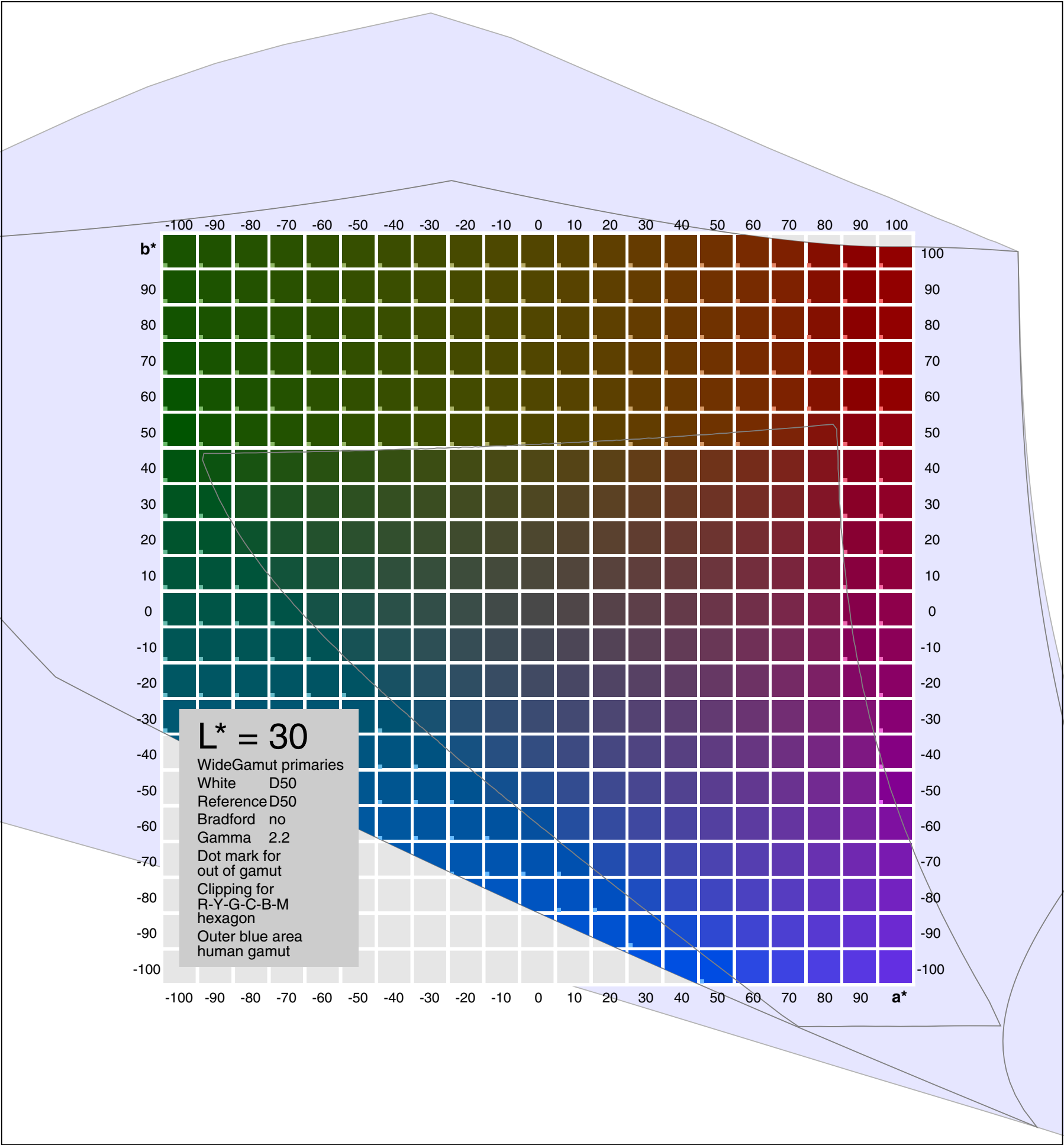
9.1 Wide Gamut RGB L*=10



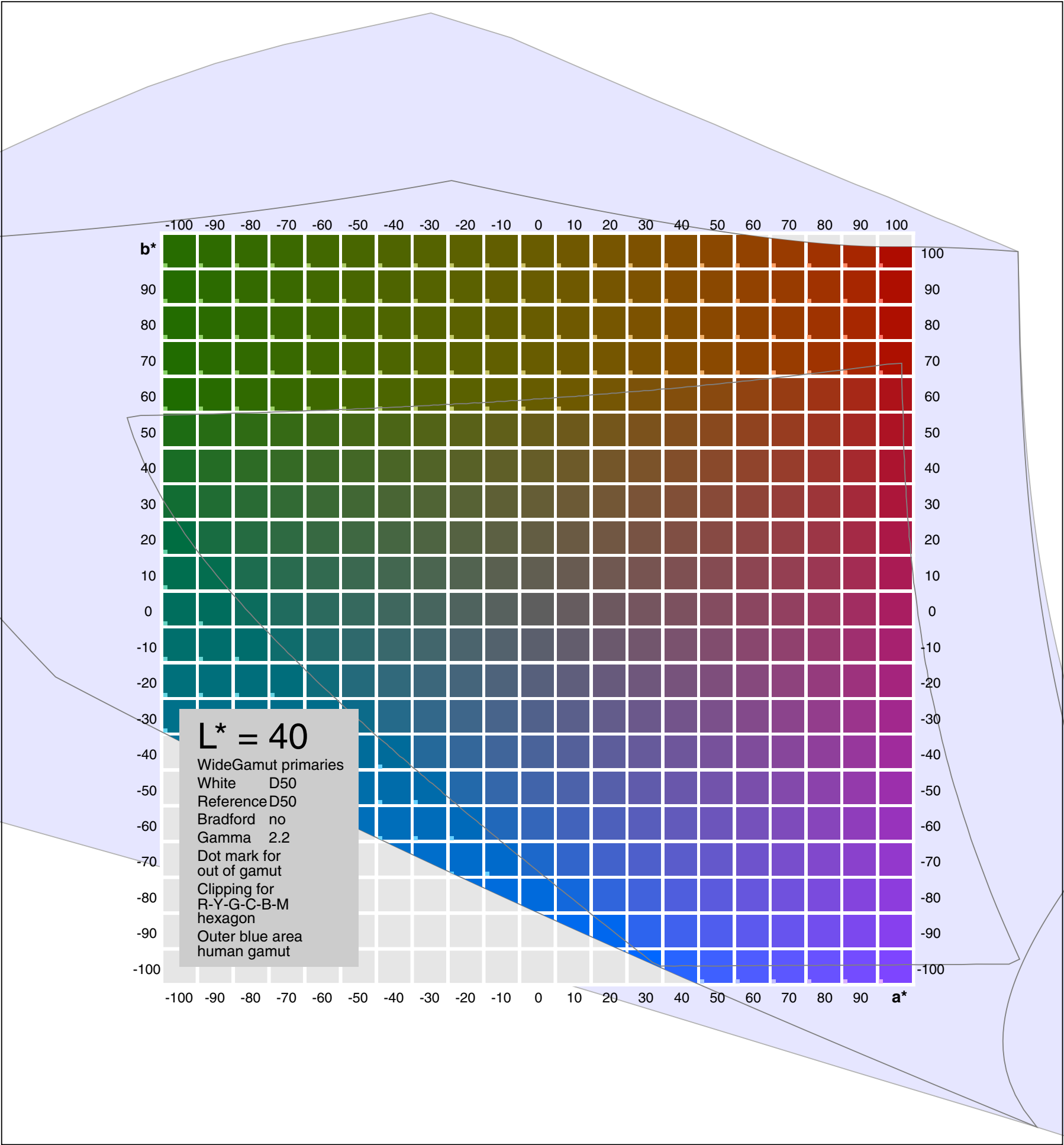
9.2 Wide Gamut RGB L*=20



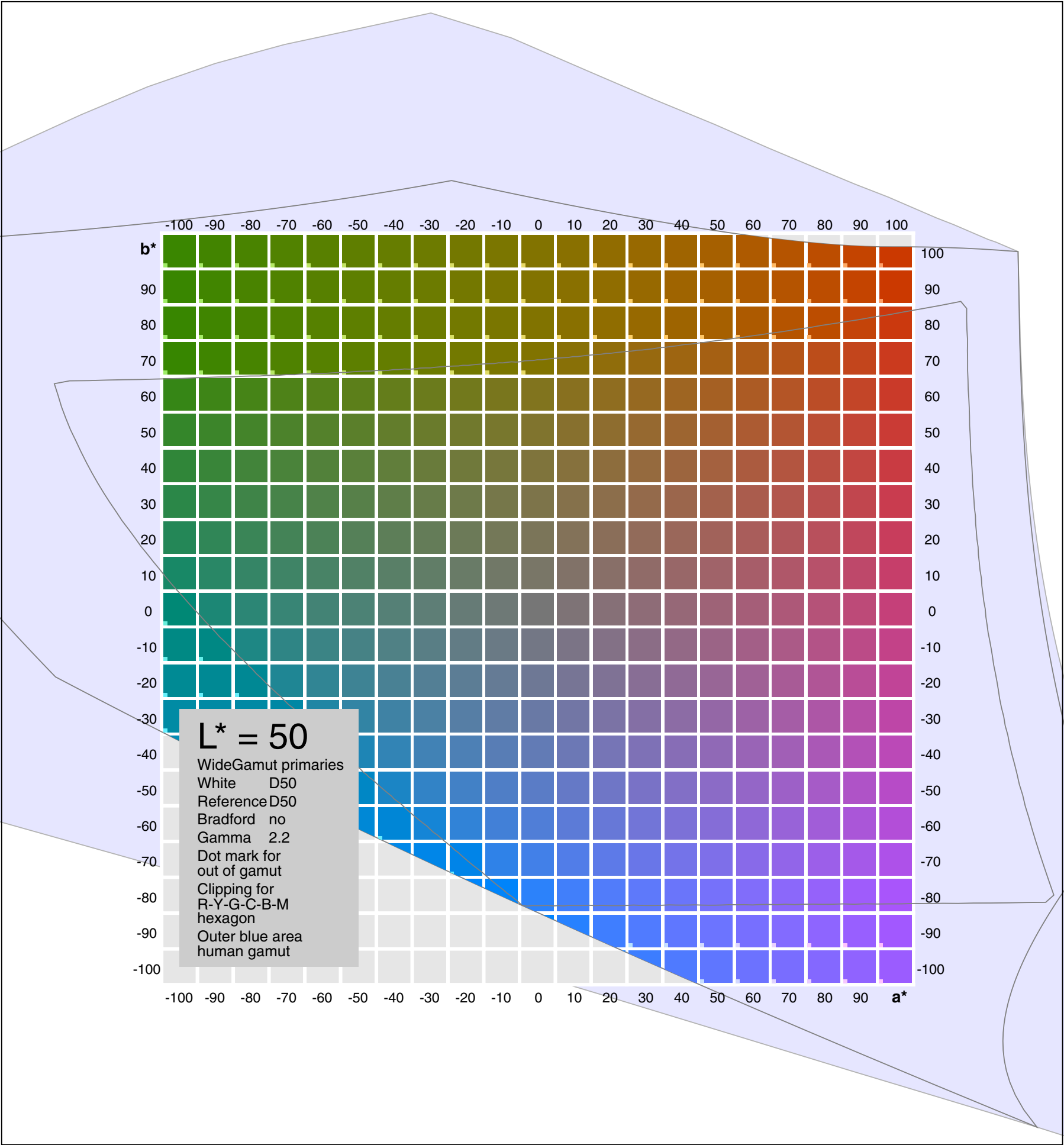
9.3 Wide Gamut RGB L*=30



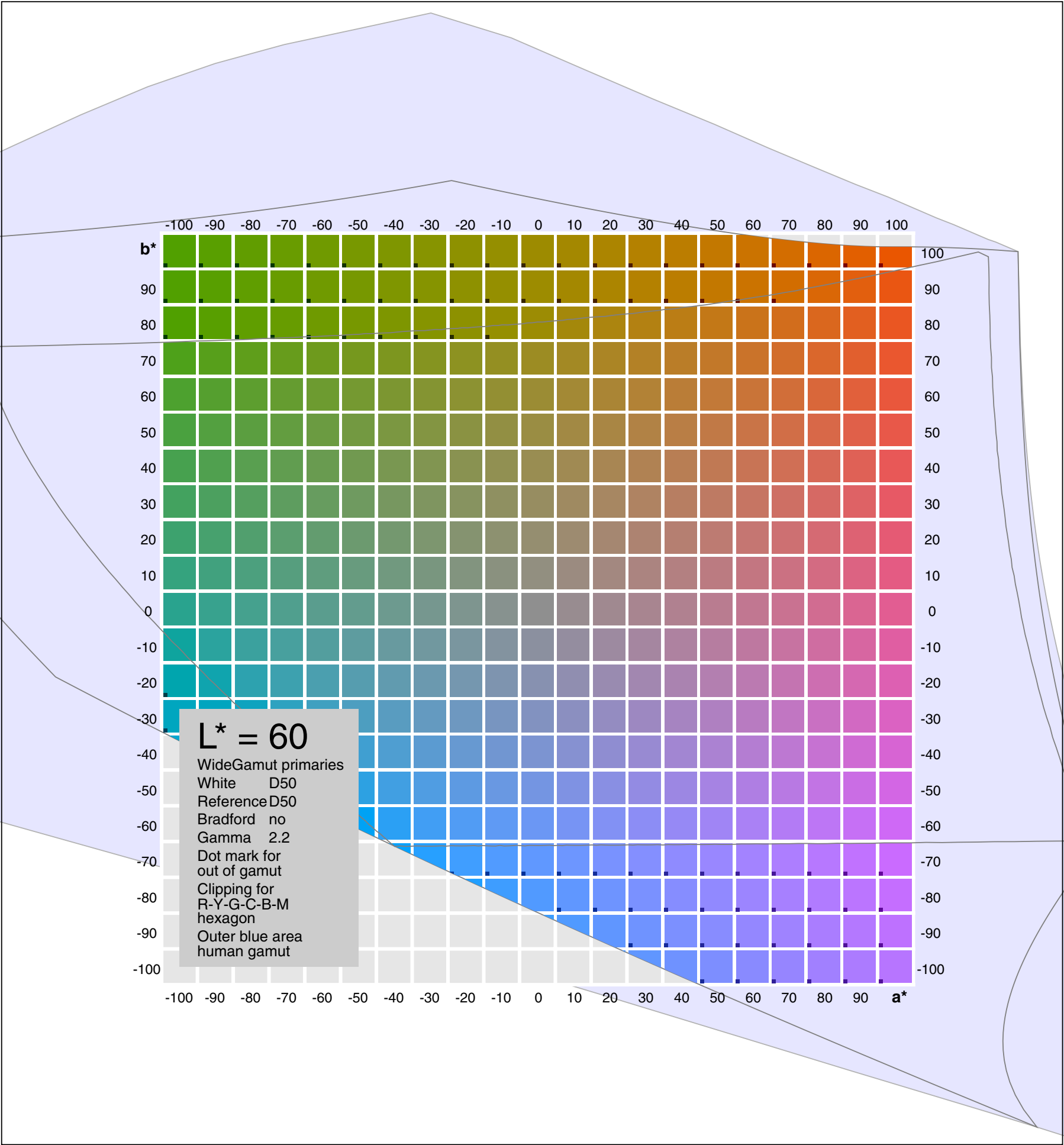
9.4 Wide Gamut RGB $L^*=40$



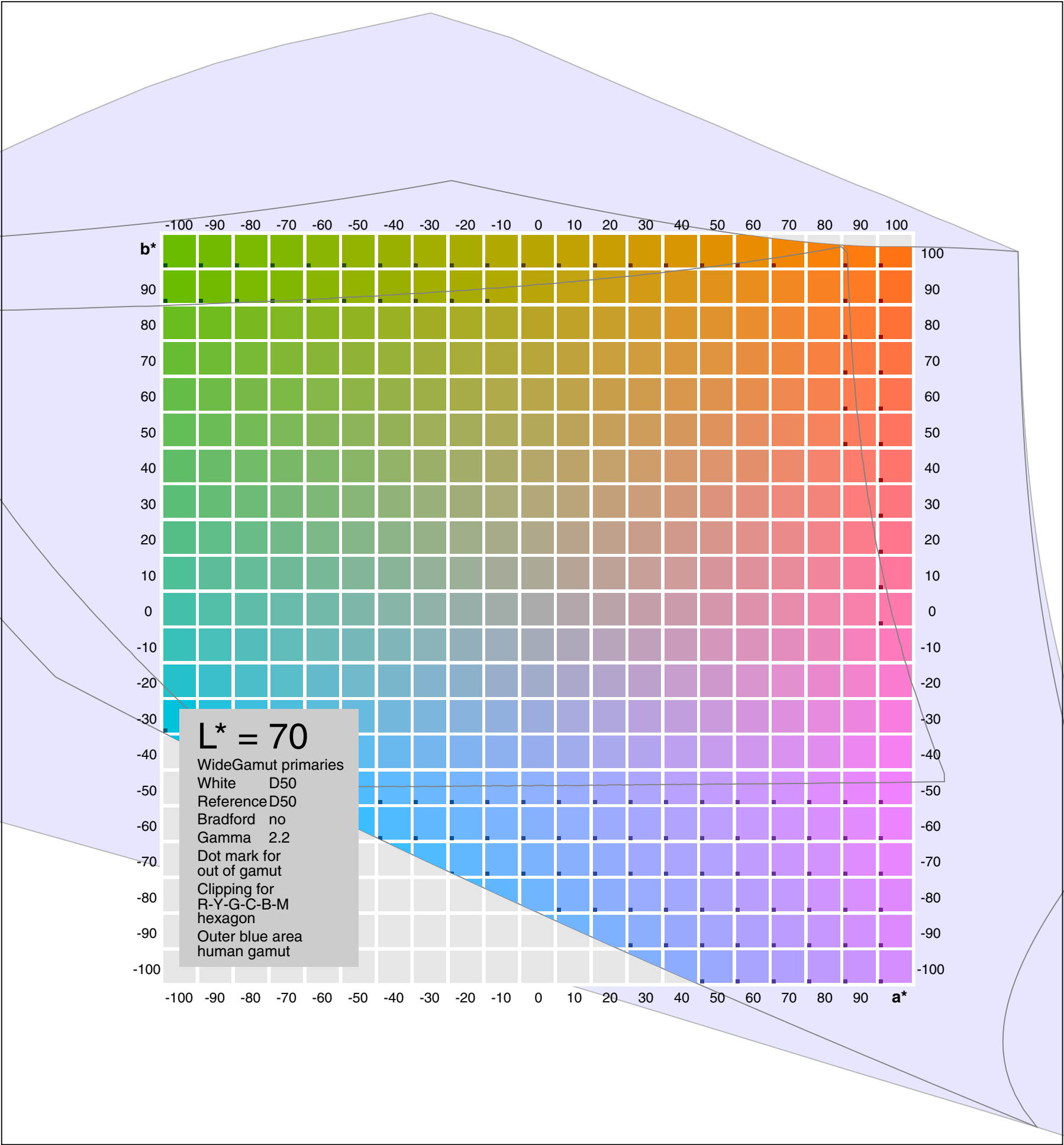
9.5 Wide Gamut RGB L*=50



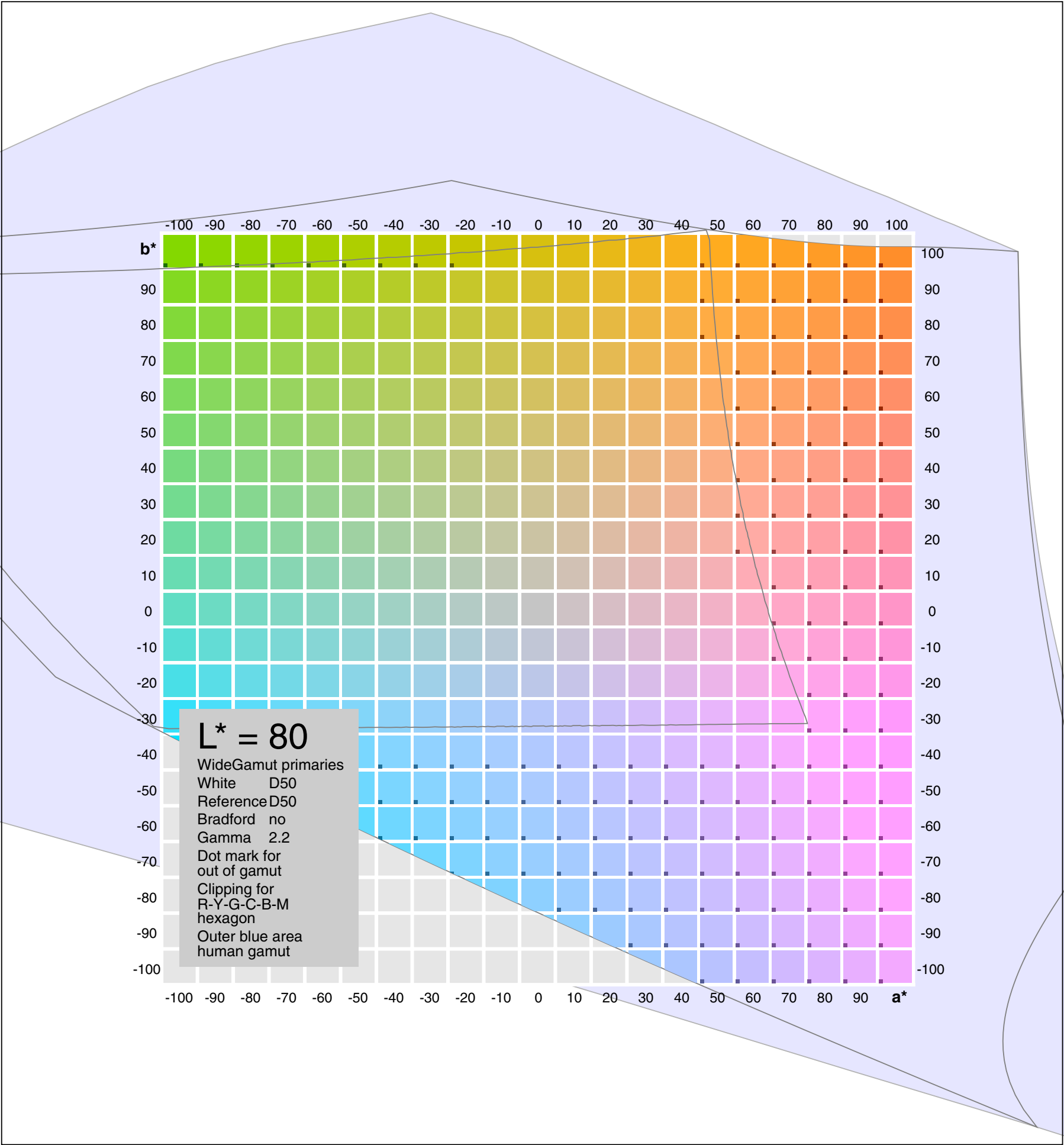
9.6 Wide Gamut RGB L*=60



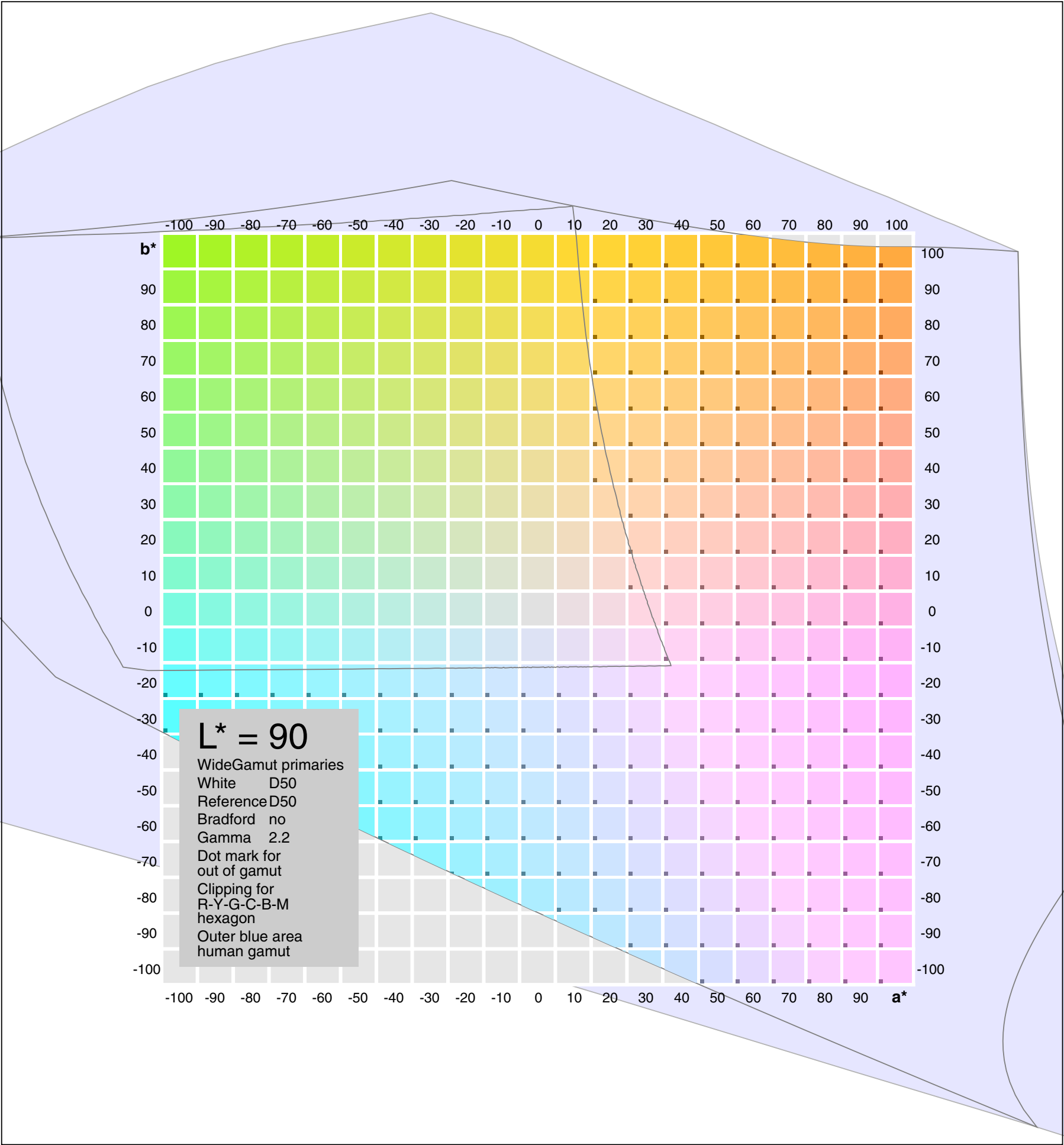
9.7 Wide Gamut RGB L*=70



9.8 Wide Gamut RGB L*=80

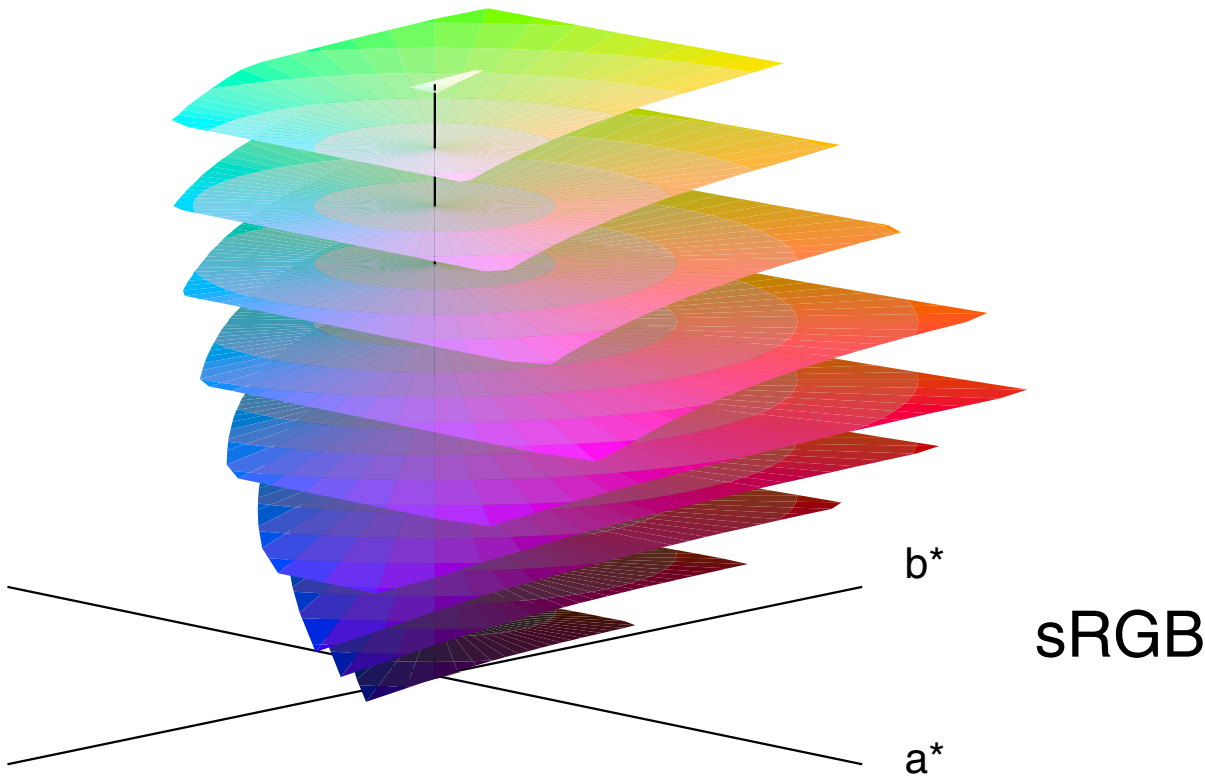
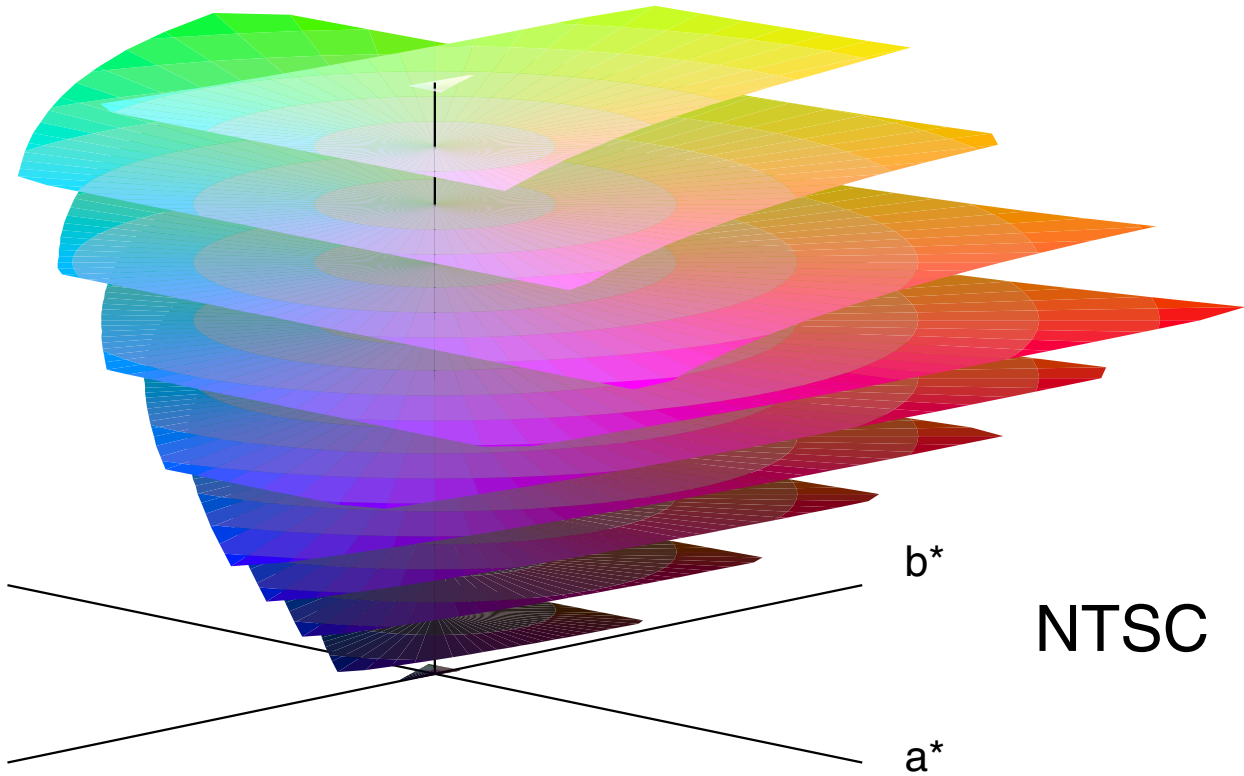
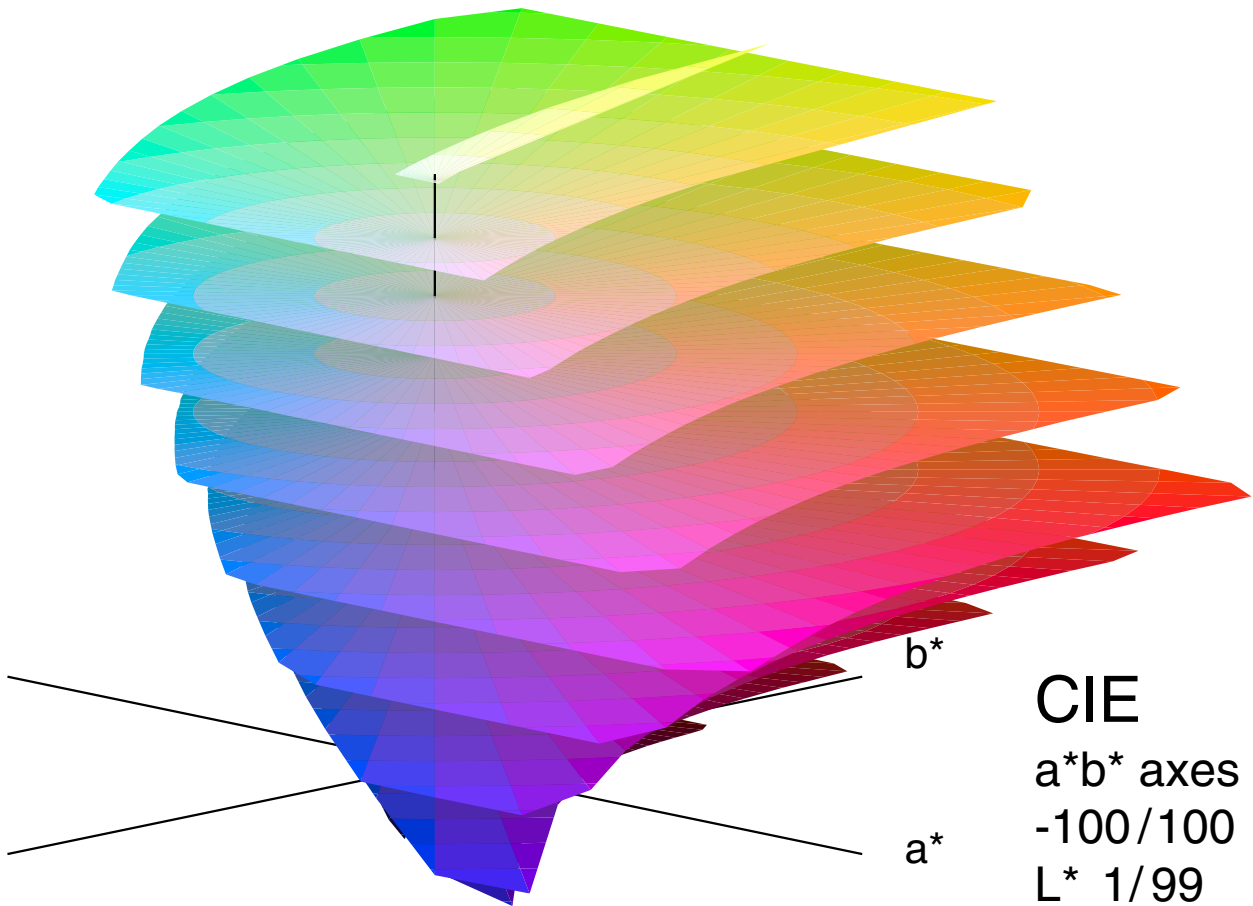


9.9 Wide Gamut RGB L*=90



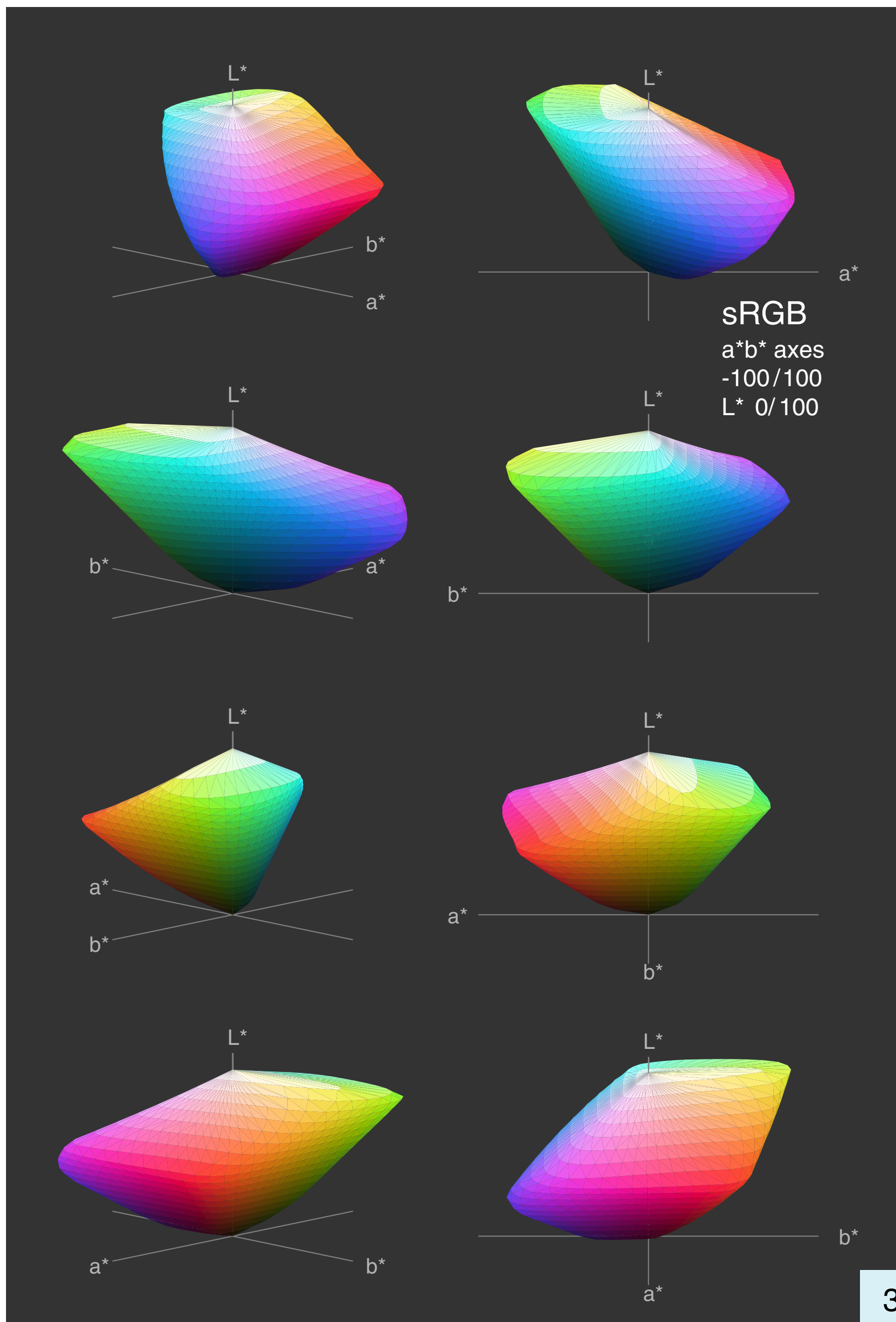
10.1 3D Views for three RGB-Spaces

No artefacts:
Smooth Line Art
Off

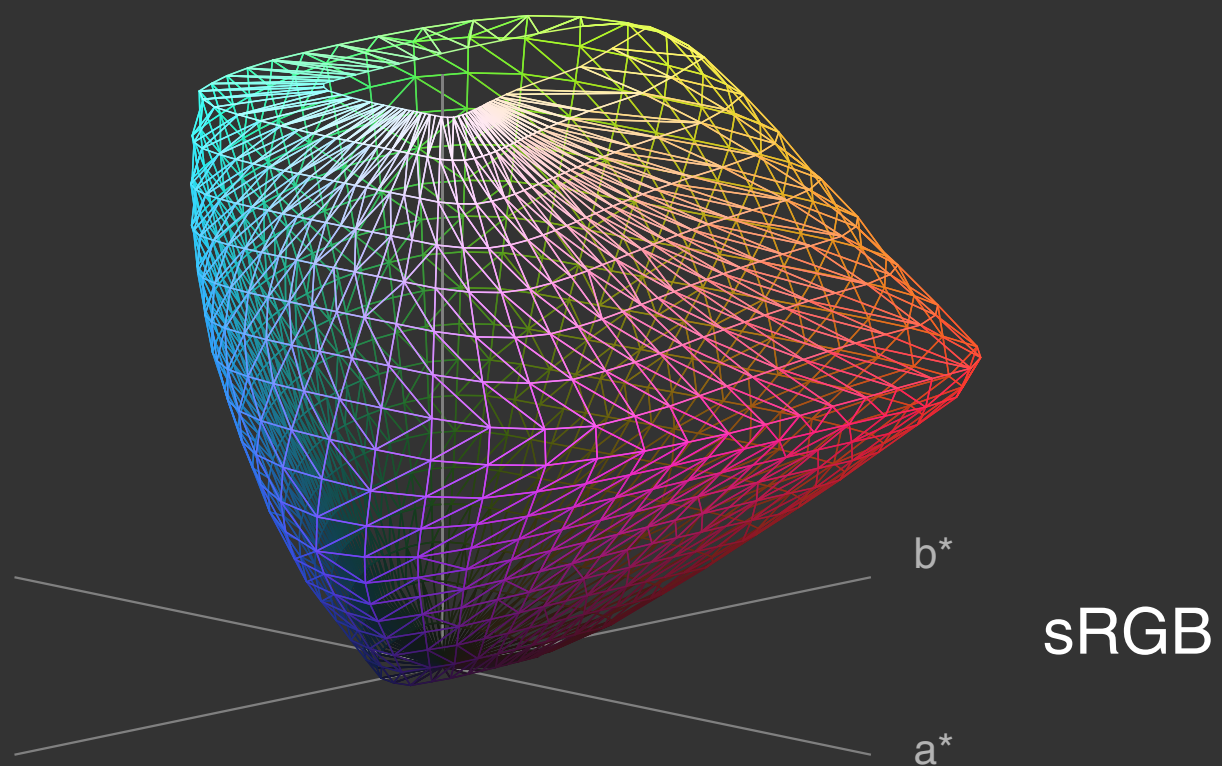
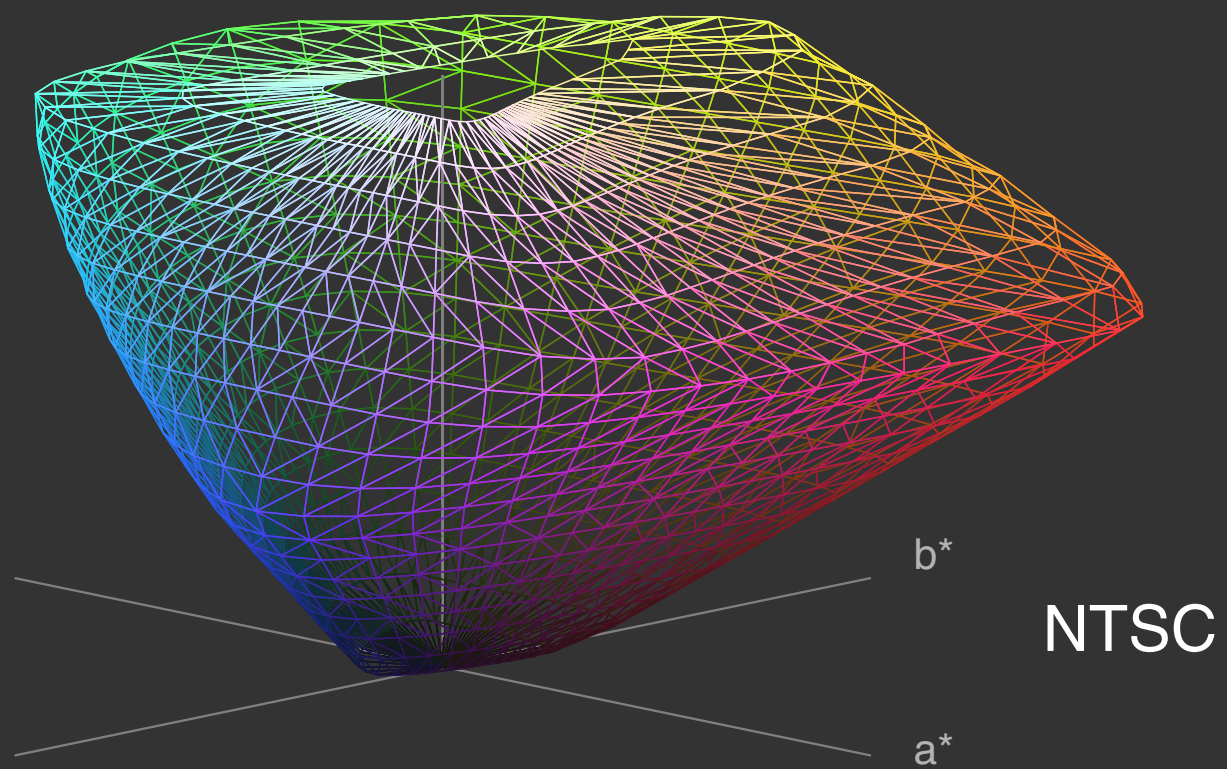
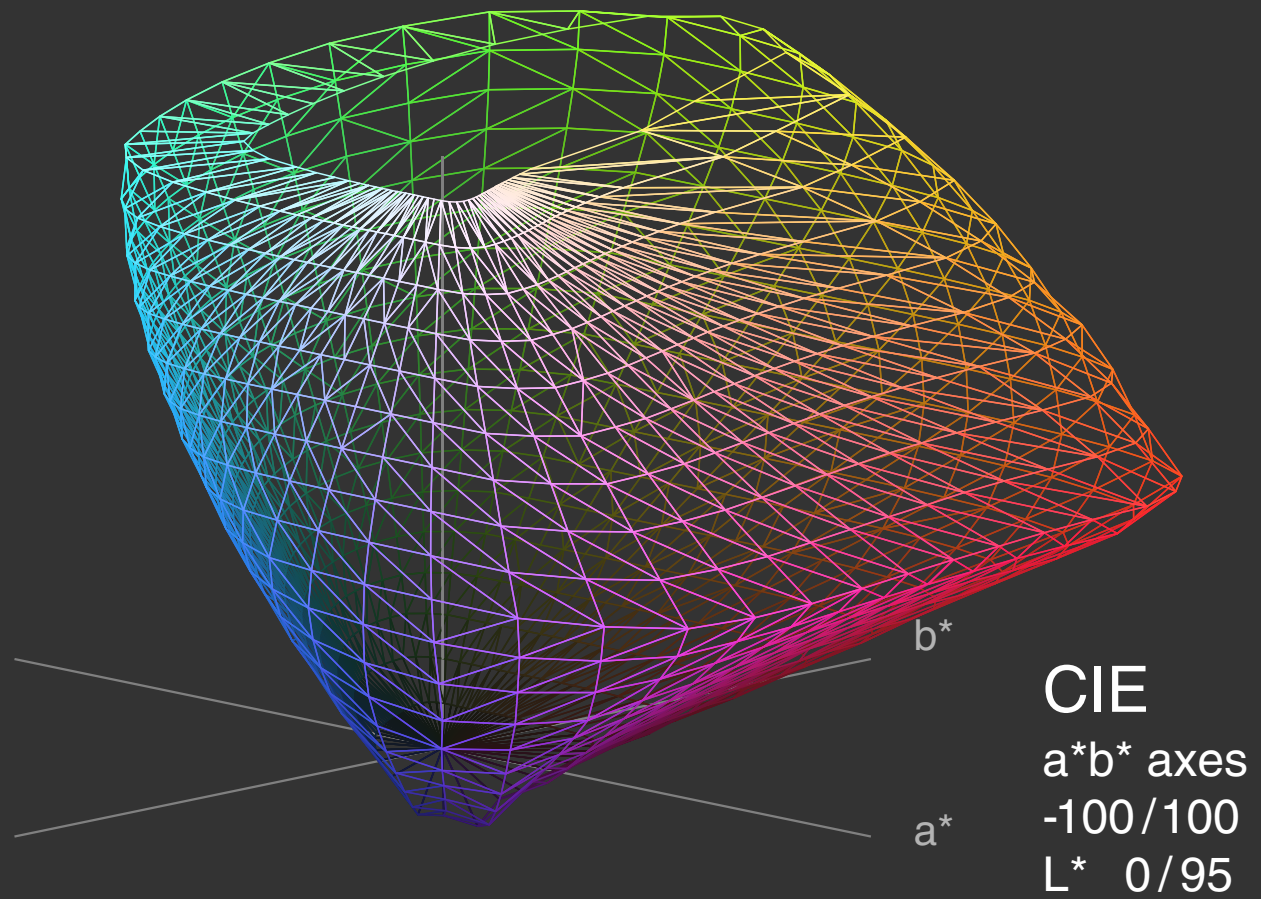


10.2 3D Views, Rotated, for sRGB

Volumes and Wireframes use smoothed height contours



10.3 3D Views

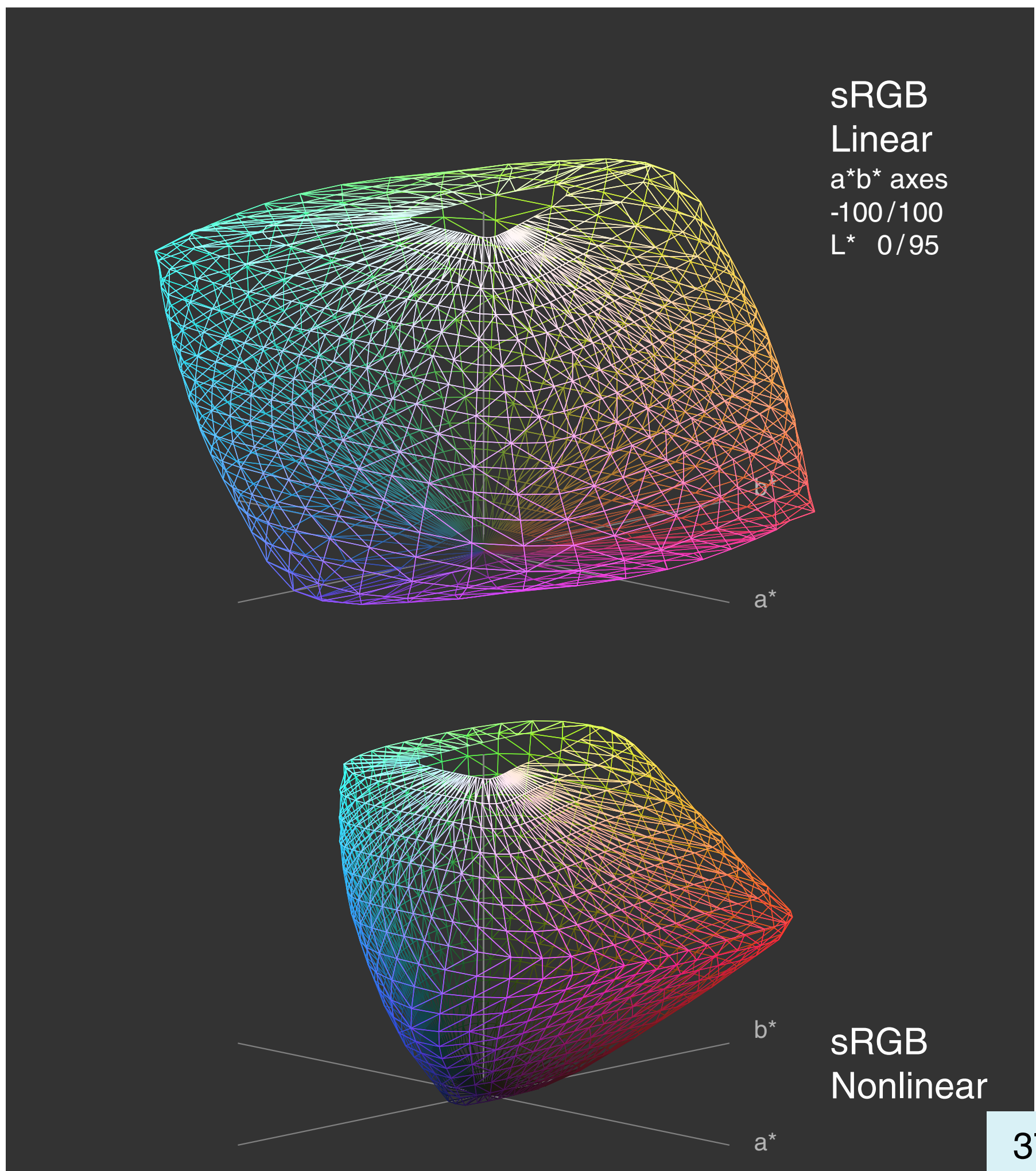


11. Linear and Standard Nonlinear CIELab

The bottom image shows the same 3D view for the sRGB color system as on the previous page. The top image shows a linear transformation, as in ch. 2.4:

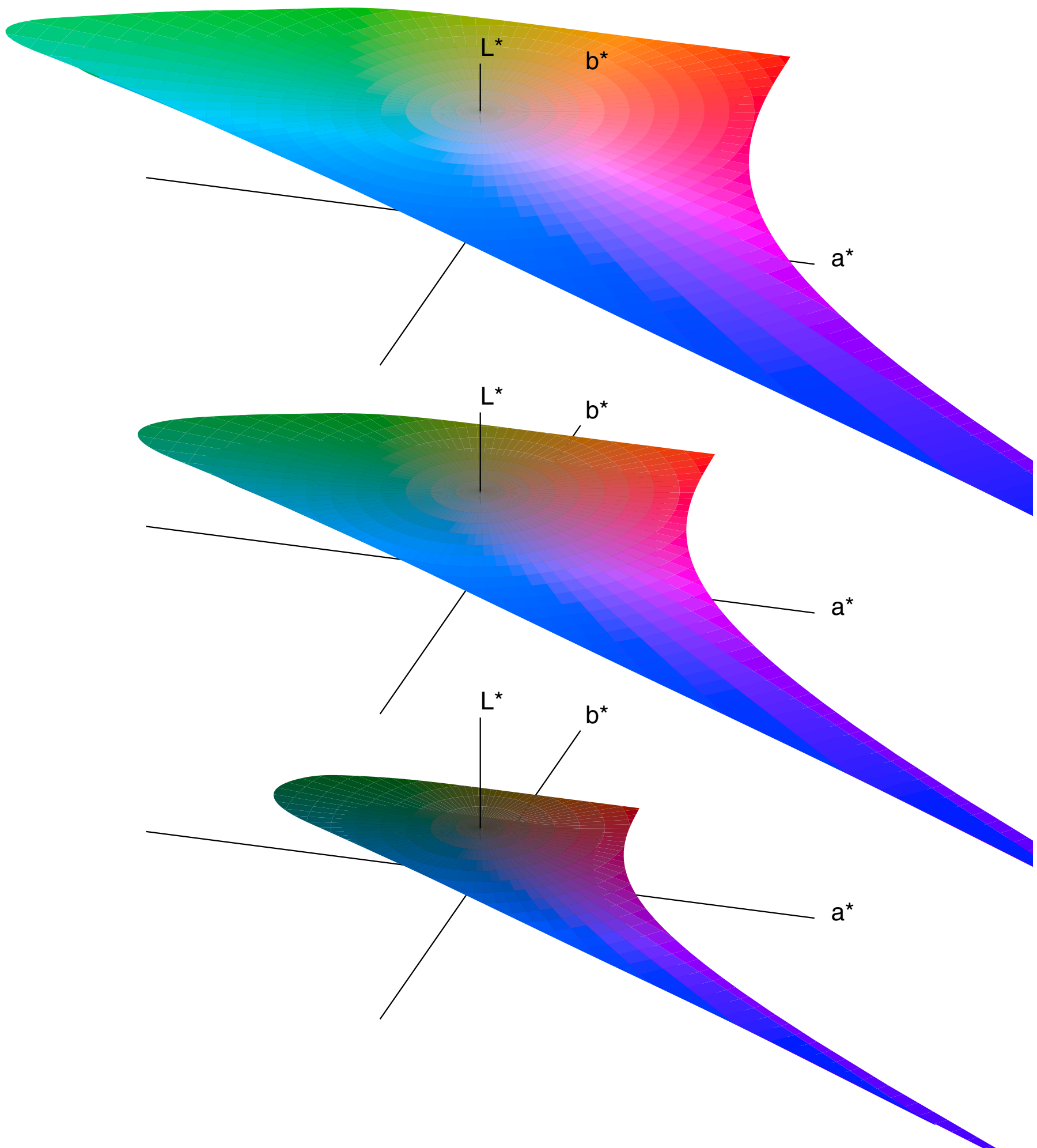
$$\begin{aligned}L^* &= 100 Y/Y_n \\ a^* &= 500 (X/X_n - Y/Y_n) \\ b^* &= 200 (Y/Y_n - Z/Z_n)\end{aligned}$$

Here we can see that the colors are too light for low luminance. This is the reason for the introduction of the nonlinear CIELab conversion, though the cubic root law is also not perfect.

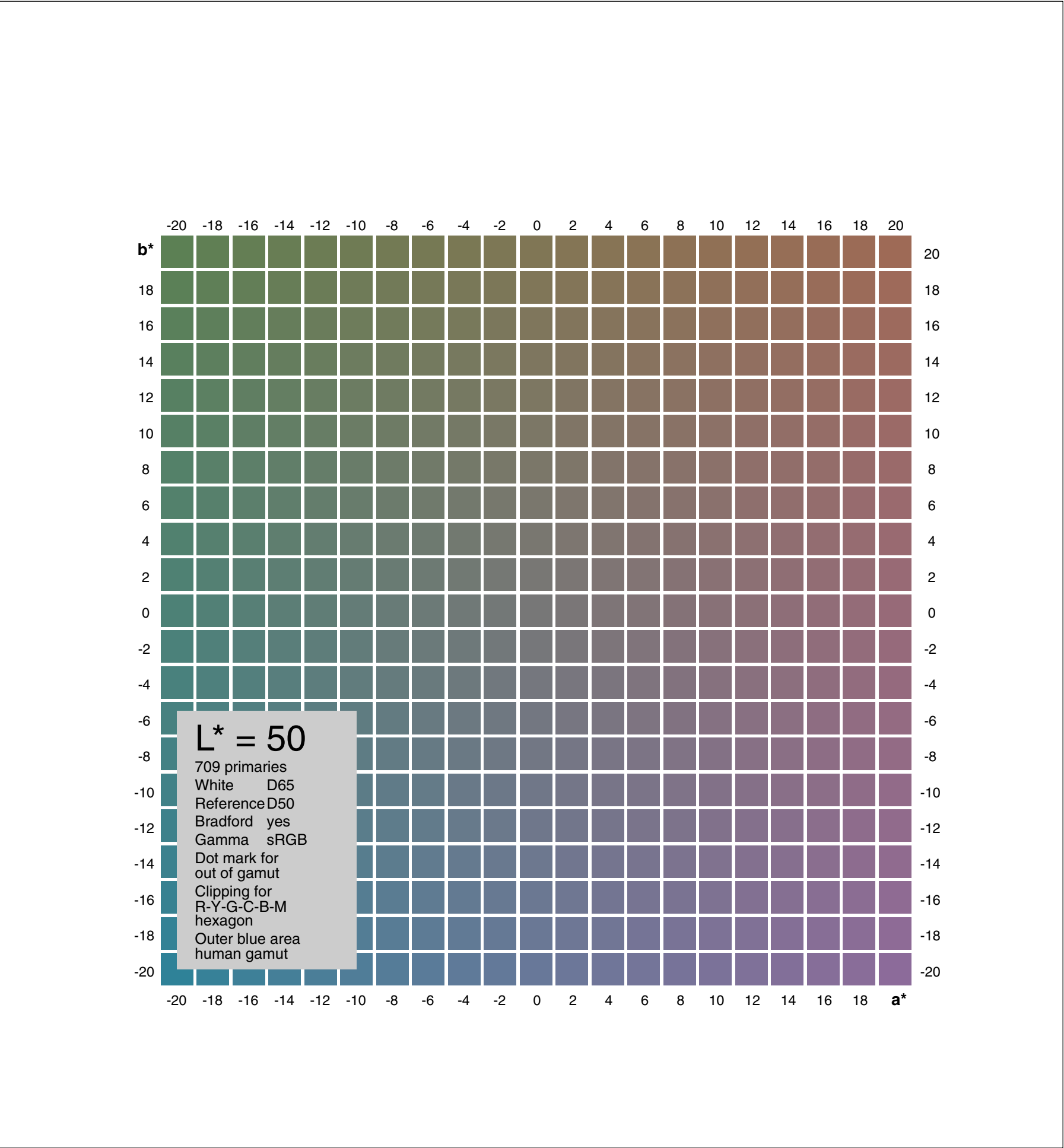


12. Human Gamut in CIELab

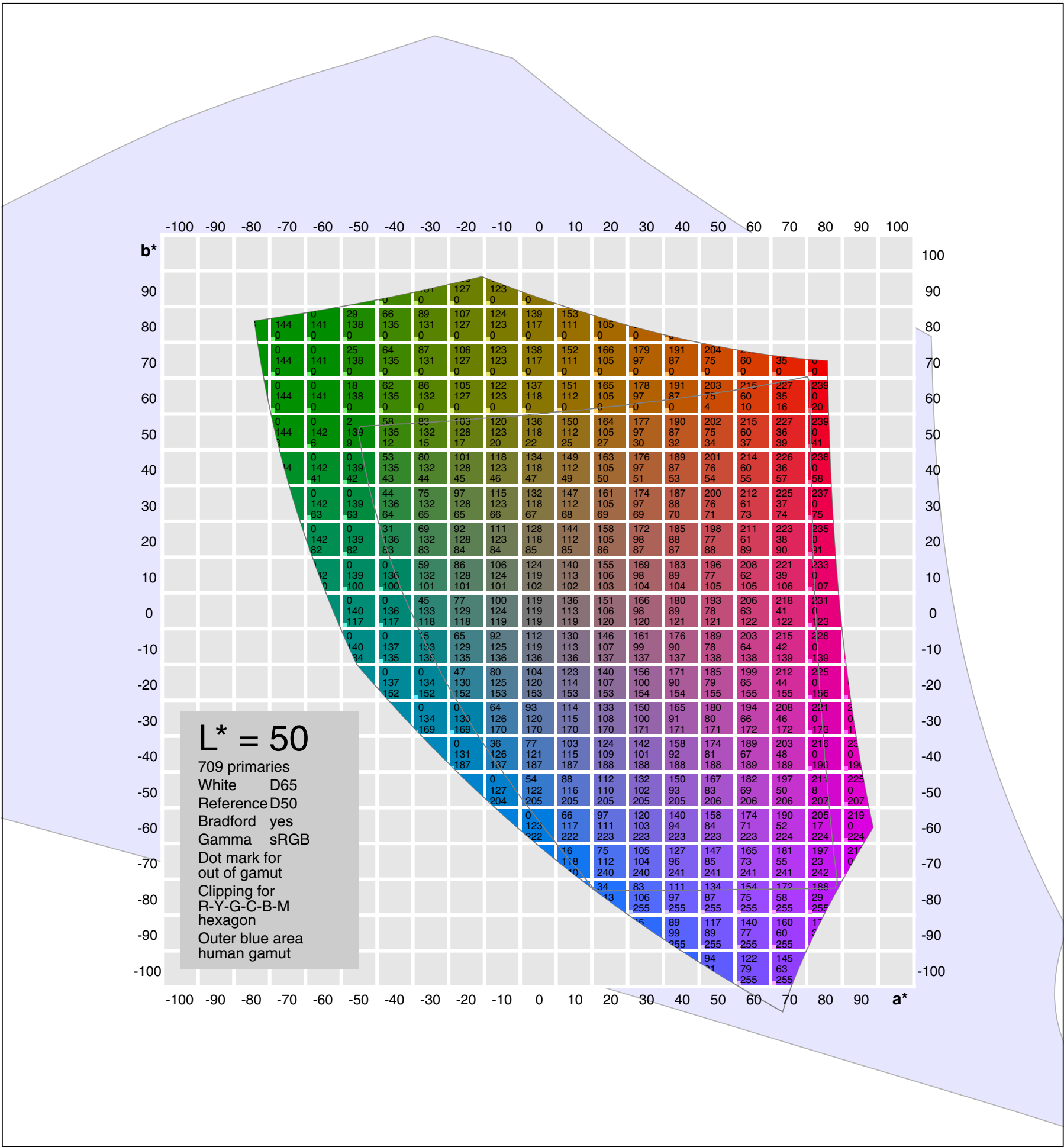
The human gamut in CIELab is a cone with the apex at the origin of the coordinate system. A threedimensional illustration can be found in [2] as well. Here we see three cross sections at $L^*=20, 40$ and 60 . The axes a^* and b^* are drawn with lengths ± 200 .



13. Low Chromaticity for sRGB L*=50



14. sRGB L*=50 with RGB Numbers



15.1 PostScript Kernels

```
/c0 1 3 div def
/c1 0.008856 def
/c2 7.787 def
/c3 16 116 div def
/c4 0.206893 def

/BM11 1.0479 def /BM12 0.0229 def /BM13 -0.0502 def
/BM21 0.0296 def /BM22 0.9904 def /BM23 -0.0171 def
/BM31 -0.0092 def /BM32 0.0151 def /BM33 0.7519 def

/PrimsCie
{/txtP (CIE primaries) def
/xr 0.73467 def /yr 0.26533 def
/xg 0.27376 def /yg 0.71741 def
/xb 0.16658 def /yb 0.00886 def
} def

/Prims709
{/txtP (709 primaries) def
/xr 0.64000 def /yr 0.33000 def
/xg 0.30000 def /yg 0.60000 def
/xb 0.15000 def /yb 0.06000 def
} def

/PrimsNTSC
{/txtP (NTSC primaries) def
/xr 0.67000 def /yr 0.33000 def
/xg 0.21000 def /yg 0.71000 def
/xb 0.14000 def /yb 0.08000 def
} def

/WhiteEqE
{/xw 0.3333 def /yw 0.3333 def
/txtW (Eq.Energy ) def
} def
/WhiteD65
{/xw 0.3127 def /yw 0.3290 def
/txtW (D65) def
} def
/WhiteNTSC
{/xw 0.3100 def /yw 0.3160 def
/txtW (NTSC) def
} def
/White3000K % R.W.G.Hunt, Measuring Colours
{/xw 0.4368 def /yw 0.4041 def
/txtW (3000K) def
} def
/White4000K
{/xw 0.3804 def /yw 0.3767 def
/txtW (4000K) def
} def
/White5000K
{/xw 0.3457 def /yw 0.3585 def
/txtW (5000K) def
} def
/White6000K
{/xw 0.3220 def /yw 0.3318 def
/txtW (6000K) def
} def
/White7000K
{/xw 0.3063 def /yw 0.3165 def
/txtW (7000K) def
} def
/White8000K
{/xw 0.2952 def /yw 0.3048 def
/txtW (8000K) def
} def
/White9300K
{/xw 0.2857 def /yw 0.2941 def % Wyszecki+Stiles interpolated
/txtW (9300K) def
} def
```

15.2 PostScript Kernels

```
/RefEqE
{/xn 0.3333 def /yn 0.3333 def
 /txtN (Eq.Energy ) def
} def
/RefD65
{/xn 0.3127 def /yn 0.3290 def
 /txtN (D65) def
} def
/RefNTSC
{/xn 0.3100 def /yn 0.3160 def
 /txtN (NTSC) def
} def
/RefD50
{/xn 0.3457 def /yn 0.3585 def
 /txtN (D50) def
} def

/PrimToMat
{ % matrix CR for XYZ(WP)=CR*RGB(WP)
 /zr 1 xr sub yr sub def
 /zg 1 xg sub yg sub def
 /zb 1 xb sub yb sub def
 /zw 1 xw sub yw sub def
 /zn 1 xn sub yn sub def
 /D xr xb sub yg yb sub mul yr yb sub xg xb sub mul sub def
 /U xw xb sub yg yb sub mul yw yb sub xg xb sub mul sub def
 /V xr xb sub yw yb sub mul yr yb sub xw xb sub mul sub def
 /u U D div def
 /v V D div def
 /w 1 u sub v sub def
 % XYZ(WP)=CR(WP)*RGB(WP)
 /CR11 u xr mul yw div def /CR12 v xg mul yw div def /CR13 w xb mul yw div def
 /CR21 u yr mul yw div def /CR22 v yg mul yw div def /CR23 w yb mul yw div def
 /CR31 u zr mul yw div def /CR32 v zg mul yw div def /CR33 w zb mul yw div def
 % Bradford matrix XYZ(D50)=BM*XYZ(D65) Actual definition in main program
 % XYZ(D50)=BM*XYZ(D65)=BM*CR(D65)*RGB(D65)=XR(D50)*RGB(D65)
 Brad
}% XR(D50)=BM*CR(D65)
/XR11 BM11 CR11 mul BM12 CR21 mul add BM13 CR31 mul add def
/XR12 BM11 CR12 mul BM12 CR22 mul add BM13 CR32 mul add def
/XR13 BM11 CR13 mul BM12 CR23 mul add BM13 CR33 mul add def
/XR21 BM21 CR11 mul BM22 CR21 mul add BM23 CR31 mul add def
/XR22 BM21 CR12 mul BM22 CR22 mul add BM23 CR32 mul add def
/XR23 BM21 CR13 mul BM22 CR23 mul add BM23 CR33 mul add def
/XR31 BM31 CR11 mul BM32 CR21 mul add BM33 CR31 mul add def
/XR32 BM31 CR12 mul BM32 CR22 mul add BM33 CR32 mul add def
/XR33 BM31 CR13 mul BM32 CR23 mul add BM33 CR33 mul add def }
}% XR(D65)=CR(D65)
/XR11 CR11 def /XR12 CR12 def /XR13 CR13 def
/XR21 CR21 def /XR22 CR22 def /XR23 CR23 def
/XR31 CR31 def /XR32 CR32 def /XR33 CR33 def } ifelse
% Inverse of XR
/D XR22 XR33 mul XR23 XR32 mul sub XR11 mul
 XR21 XR33 mul XR23 XR31 mul sub XR12 mul sub
 XR21 XR32 mul XR22 XR31 mul sub XR13 mul add def
/RX11 XR22 XR33 mul XR23 XR32 mul sub D div def
/RX12 XR12 XR33 mul XR13 XR32 mul sub D div neg def
/RX13 XR12 XR23 mul XR13 XR22 mul sub D div def
/RX21 XR21 XR33 mul XR23 XR31 mul sub D div neg def
/RX22 XR11 XR33 mul XR13 XR31 mul sub D div def
/RX23 XR11 XR23 mul XR13 XR21 mul sub D div neg def
/RX31 XR21 XR32 mul XR22 XR31 mul sub D div def
/RX32 XR11 XR32 mul XR12 XR31 mul sub D div neg def
/RX33 XR11 XR22 mul XR12 XR21 mul sub D div def
/Yw 1 def
/Xw xw yw div Yw mul def
/Zw zw yw div Yw mul def
/Yn 1 def
/Xn xn yn div Yn mul def
/Zn zn yn div Yn mul def
} bind def
```

15.3 PostScript Kernels

```
/LABtoRGB
{ /Y1 L* 0.16 add 1.16 div def
  /X1 a* 5.0 div Y1 add def
  /Z1 Y1 b* 2.0 div sub def
  /X X1 c4 le {X1 c3 sub c2 div }{ X1 3 exp } ifelse Xn mul def
  /Y Y1 c4 le {Y1 c3 sub c2 div }{ Y1 3 exp } ifelse Yn mul def
  /Z Z1 c4 le {Z1 c3 sub c2 div }{ Z1 3 exp } ifelse Zn mul def
  /R X RX11 mul Y RX12 mul add Z RX13 mul add def
  /G X RX21 mul Y RX22 mul add Z RX23 mul add def
  /B X RX31 mul Y RX32 mul add Z RX33 mul add def
} bind def

/RGBtoLAB
{%/c0 1 3 div def Actual definition in main program
%/c1 0.008856 def
%/c2 7.787 def
%/c3 16 116 div def
%/c4 0.206893 def
/X R XR11 mul G XR12 mul add B XR13 mul add def
/Y R XR21 mul G XR22 mul add B XR23 mul add def
/Z R XR31 mul G XR32 mul add B XR33 mul add def
/X1 X Xn div dup c1 le { c2 mul c3 add }{ c0 exp } ifelse def
/Y1 Y Yn div dup c1 le { c2 mul c3 add }{ c0 exp } ifelse def
/Z1 Z Zn div dup c1 le { c2 mul c3 add }{ c0 exp } ifelse def
/a* X1 Y1 sub 5.0 mul def
/b* Y1 Z1 sub 2.0 mul def
/ga* a* 1.05 add 100 mm mul def
/gb* b* 1.05 add 100 mm mul def
} bind def

/IGamma
{ /iga 1 gam div def
  /og false def % out of gamut
  R 0 lt { /R 0 def /og true def } if
  G 0 lt { /G 0 def /og true def } if
  B 0 lt { /B 0 def /og true def } if
  R 1 gt { /R 1 def /og true def } if
  G 1 gt { /G 1 def /og true def } if
  B 1 gt { /B 1 def /og true def } if
  gam 2.4 ne
  {/R R iga exp def
  /G G iga exp def
  /B B iga exp def }
  {/R R dup 0.00304 lt {12.92 mul }{iga exp 1.055 mul 0.055 sub } ifelse def
  /G G dup 0.00304 lt {12.92 mul }{iga exp 1.055 mul 0.055 sub } ifelse def
  /B B dup 0.00304 lt {12.92 mul }{iga exp 1.055 mul 0.055 sub } ifelse def } ifelse
  R G B setrgbcolor
} bind def

/xyYtoLAB
% input x,y,Y
{ /z 1 x sub y sub def
  /X x y div Y mul def
  /Z z y div Y mul def
  /R X RX11 mul Y RX12 mul add Z RX13 mul add def
  /G X RX21 mul Y RX22 mul add Z RX23 mul add def
  /B X RX31 mul Y RX32 mul add Z RX33 mul add def
  /max R def
  G max gt {/max G def } if
  B max gt {/max B def } if
  /R R max div def
  /G G max div def
  /B B max div def
  RGBtoLAB
} bind def
```

16. References

- [1] R.W.G.Hunt
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Fountain Press England
1998
 - [2] G.Wyszecki + W.S.Stiles
Color Science
John Wiley & Sons, New York ,..., 1982
 - [3] References for Color Science
<http://www.fho-emden.de/~hoffmann/colcie290800.pdf>
 - [4] References for PostScript
<http://www.fho-emden.de/~hoffmann/pstutor22112002.pdf>
 - [5] Everything about Color and Computers
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 - [6] M.Nielsen + M.Stokes
The Creation of the sRGB ICC Profile
<http://www.srgb.com/c55.pdf>
Year unknown, after 1998
 - [7] International Color Consortium
<http://www.color.org>
 - [8] Specification ICC.1:21001-12
File Format for Color Profiles (Version 4.0.0)
<http://www.color.org/newiccspec.pdf>
- This doc:
<http://www.fho-emden.de/~hoffmann/cielab03022003.pdf>